Tentamen: EI1120 Elkretsanalys (CENMI), 2013-03-14, kl 08–13

Hjälpmedel: *Ett* A4-ark med studentens anteckningar (båda sidor). Dessutom, pennor!

Tentan har 7 tal: 2 i del A (10p), 2 i del B (12p) och 3 i del C (18p).
Obs: Samma tal står här först **på engelska** (s.1–s.3) och sedan **på svenska** (s.4–s.6).
Du får välja mellan dessa språk för svaren.

Läs varje tal noggrant **innan du försöker svara**.
Tänk på att **använda återstående tid till att kolla på varje svar**: man kan göra dimensionsanalys, rimlighetsbedömning (t.ex. ”är det rätt att y går ner medan x går ner?”), och lösning genom en alternativ metod. Lösningar ska **förenklas** om inte annat är specificerat.
Var försiktig med att **inte satsa för mycket tid** på bara en uppgift om du fastnar: ta hänsyn till poängvärden av uppgifterna, och att man måste klara varje del av tentan. Det är ofta så att **senare** deltal är betydligt **svårare** än de första deltalen.

Godkänt vid ≥50% på del A, ≥25% på del B och på C, och ≥50% på delar B och C tillsammans.
Godkänd kontrollskrivning gör att man redan klarat del A här på tentan.
Betyget räknas från B och C delarna: det finns därför ingen fördel med att svara på A-delen om man har godkänd KS, eller att försöka få mycket hög poäng i A-delen.
Eventuella bonuspoäng från KS och hemuppgifter tillkommer enligt KursPM. Se också PM:et angående rättningsnormer och överklagande. Instruktionerna ovan tar prioritet över PM vid skillnad (t.ex. hjälpmedel).

Examinator: Nathaniel Taylor

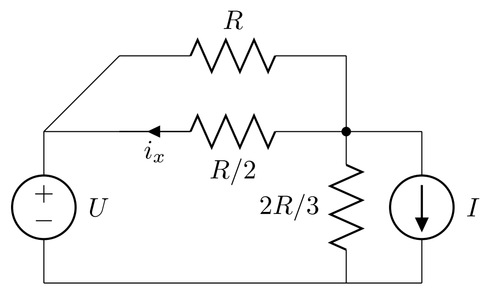
In English

Part A. DC (static solutions). NOT NEEDED IF KS PASSED: SEE NOTES ABOVE.

1) [5p]

The current source I has value $I = \frac{3U}{4R}$.
Solve for the current marked i_x, in the resistor of value $R/2$.
Answer in terms of the known quantities U and R.

Suggestion: Two simplifications can be used to reach a single loop of two voltage sources and two resistors, that is easily solved by KVL or by further simplifications to one source and resistor; then current division can be used to find i_x. You are however free to solve by any method you like. Remember you are advised even to check the solution by another method if you have time.

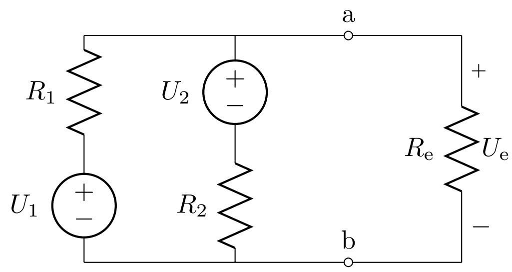


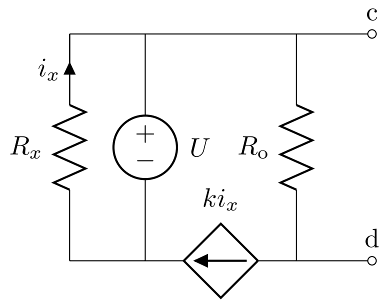
2) [5p]

a) [2p] In the *upper* circuit on the right, find the Thevenin equivalent between nodes a and b, for the part of the circuit to the left of those nodes (i.e. only the components U_1, R_1, U_2 and R_2). Only the component values are known: U_e is unknown.

b) [2p] Now consider the entire upper circuit, *including* R_e. Find the voltage U_e across R_e. Your answer to part ‘a)’ might be helpful.

c) [1p] For the *lower* circuit, find the Norton equivalent between the nodes c and d. The dependent current source is controlled by the marked current i_x, which is unknown.



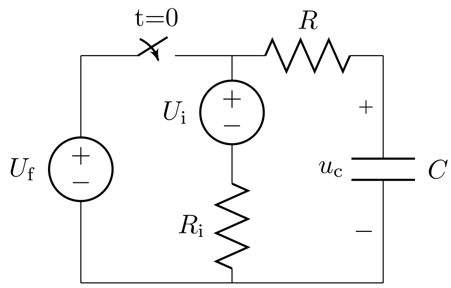


Part B. Transient analysis

3) [6p]

Find $u_c(t)$ for all time $t > 0$.
Note that the switch turns on (becomes a short-circuit) at $t = 0$.

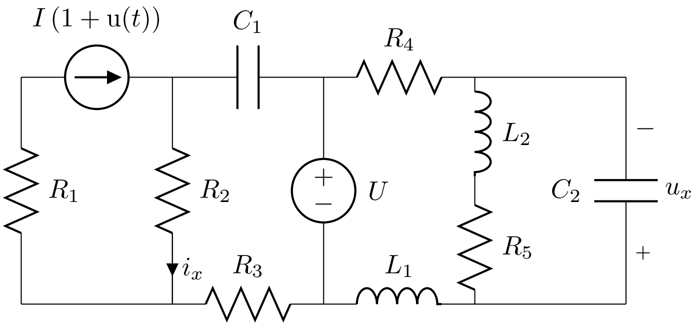
(Assume, of course, that the circuit has reached an equilibrium before $t = 0$.)



4) [6p]

a) [4p] Find u_x and i_x *before* time $t = 0$, i.e. the equilibrium when the current source has a value of just I.

b) [2p] Find u_x and i_x just *after* time $t = 0$, i.e. "$t = 0^+$" when the current source has just changed its current but the stored energies in reactive components (L and C) have not changed.



All these components have known values; all are constant *except* the current source, which has a step-change in output at $t = 0$ (u(t) is the unit step). The quantities u_x and i_x are unknown.

Part C. AC (sinusoidal steady state)

5) [6p]

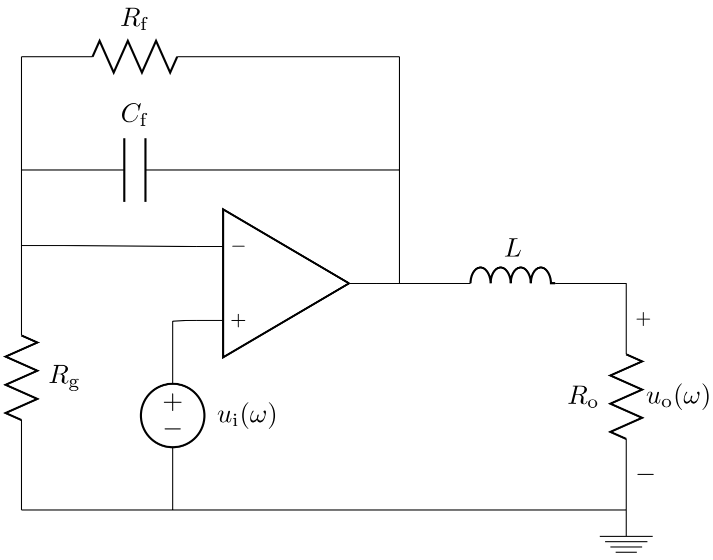
a) [2p] Sketch a bode amplitude plot (i.e. dB versus log(ω)) for this network function:

$$H(\omega) = k \frac{(1 + \mathrm{j}\omega/\omega_3)}{(1 + \mathrm{j}\omega/\omega_1)\ (1 + \mathrm{j}\omega/\omega_2)}$$

where k, ω_1, ω_2 and ω_3 are all positive real constants. Assume $\omega_1 \ll \omega_2 \ll \omega_3$. Which of the common names of 'filters' (HP,LP,BP,BS) best describes this plot?

b) [3p] For the circuit shown on the right, find the network function $H(\omega) = u_o(\omega)/u_i(\omega)$.

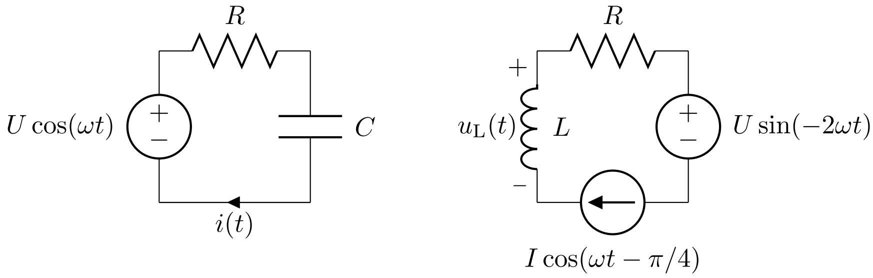
c) [1p] Show that the result for $H(\omega)$ in part 'b)' can be written in the form shown in part 'a)': express the four constants (k, ω_1, ω_2 and ω_3) in terms of the circuit components.



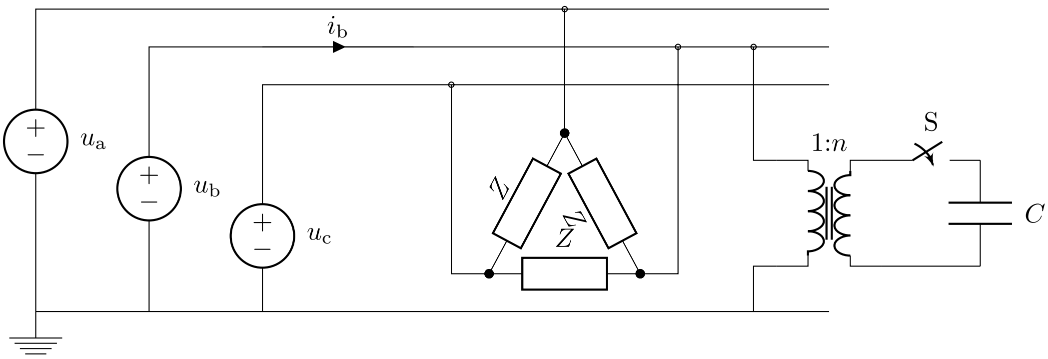
6) [6p]

a) [4p] In the circuit to the left, find $i(t)$.

b) [2p] In the circuit to the right, find $u_{\mathrm{L}}(t)$.



7) [6p]



The diagram above shows an ideal three-phase voltage-source supplying a balanced three-phase load and an unbalanced load. The source has angular frequency ω, rms phase-voltage magnitude U_{p}, and phase-sequence *abc*. We will work entirely in the frequency domain, taking u_{a} as the reference phase (0°) and using rms values. This means for example that the phasor for the c-phase voltage is $u_{\mathrm{c}} = U_{\mathrm{p}}\angle{-4\pi/3}$ (or equivalently, $U_{\mathrm{p}}\angle{2\pi/3}$).

The balanced load consists of phase-impedances $Z = R + \mathrm{j}\omega L$ in a Δ-connection. The unbalanced load is a capacitor C connected by a switch 'S' across one winding (coil) of an ideal transformer with ratio 1:n. The transformer's other winding is connected between the b-phase and neutral. (The directions of the transformer windings are not shown by dots: this information is not relevant to the question.)

Case 1: switch 'S' open (*not* connected).
Thus, only the source and the balanced load are important.

a) [1p] What is the magnitude of the line-voltage (i.e. of the voltage across each impedance Z)? You are not required to show a derivation.

b) [2p] What is the total complex power delivered from the 3-phase source?

Case 2: switch 'S' is now closed (connected) and the transformer ratio n is adjusted to give perfect power-factor correction of the current drawn from phase-b of the source. Note that phases a and c are unaffected: the transformer is connected as a single phase load.

c) [1p] What now is the total complex power delivered from the 3-phase source?

d) [1p] What is the current i_{b} (as a phasor, not just magnitude)?

e) [1p] What ratio n is needed in order to give the perfect power-factor correction of phase-b, as described above ('Case 2')? Express n in the known quantities.

END of exam. Don't forget to use remaining time to check all your answers in every imaginable way!

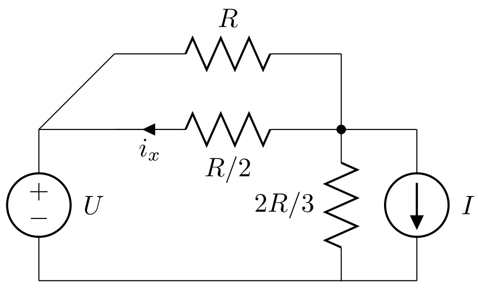
På svenska (samma problem)

Del A. Likström. BEHÖVS INTE VID GODKÄND KS: SE SIDA 1.

1) [5p]

Strömkällan I ger $I = \frac{3U}{4R}$.
Bestäm strömmen i_x, som går i resistorn $R/2$.
Ge svaret i de kända storheterna U och R.

Ledning: Kretsen kan förenklas i två steg, vilket ger en enda slinga med två spänningskällor och två resistorer, som går lätt att lösa genom KVL eller genom att den förenklas mer. Strömdelning kan sedan tillämpas för att få i_x. Men du får gärna använda en annan metod. Det rekommenderas att du verifierar ditt svar genom en alternativ metod.

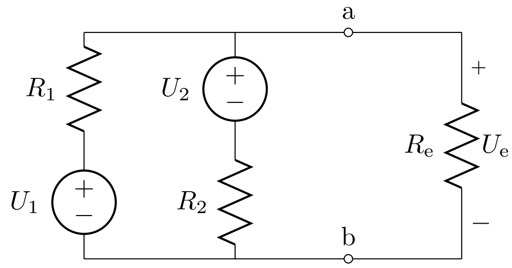


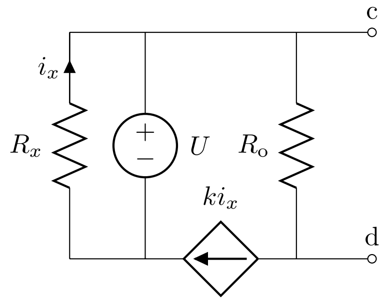
2) [5p]

a) [2p] I den *övre* kretsen till höger, bestäm Theveninekvivalenten med avseende på polparet a och b, för kretsen till vänster om polparet (d.v.s. ekvivalenten är bara för komponenterna U_1, R_1, U_2 och R_2). Bara komponentvärdena är kända (U_e är okänd).

b) [2p] Nu för den hela *övre* kretsen (*inklusive* R_e), bestäm spänningen U_e över resistorn R_e. Svaret till del 'a)' kan möjligen förenkla lösningen här.

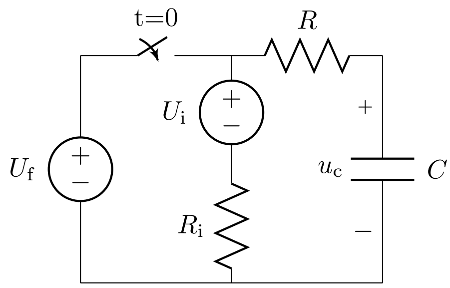
c) [1p] För den *lägre* kretsen, bestäm Nortonekvivalenten med avseende på polparet c och d. Den strömstyrdeströmkälla styrs av den markerade strömmen i_x, vilken är okänd.





Del B. Transientanalys

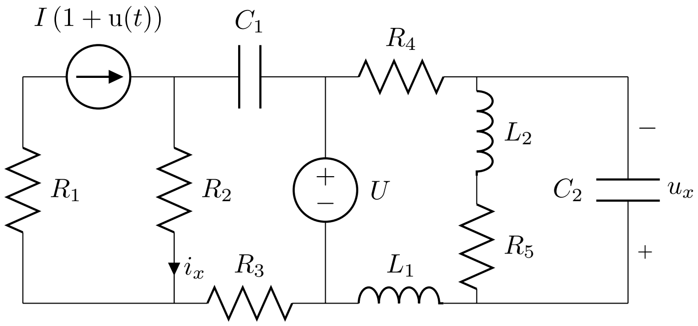
3) [6p]



Bestäm $u_{\mathrm{c}}(t)$ som funktion av tid, för tider $t > 0$.
Observera att brytaren kopplas in (blir kortslutning) vid tiden $t = 0$.

(Anta att kretsen har kommit till jämviktsläge innan $t = 0$.)

4) [6p]



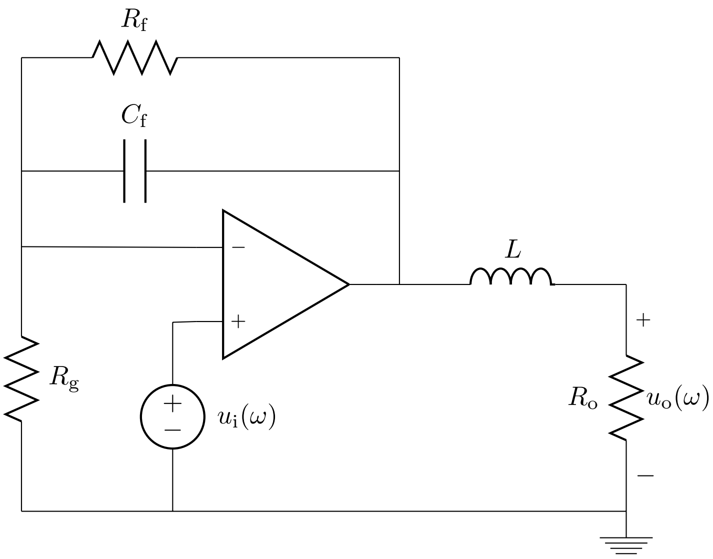
a) [4p] Bestäm u_x och i_x just *innan* tiden $t = 0$, d.v.s. jämviktsläget när strömkällan har värdet I.

b) [2p] Bestäm u_x och i_x precis *efter* tiden $t = 0$, d.v.s. "$t = 0^+$" när strömkällan har ett nytt värde av ström, men de lagrade energierna i de reaktiva komponenterna (L och C) har inte hunnit ändras.

Information: Alla komponentvärdena är kända, och alla är konstanta *förutom* strömkällan, vilken har en diskret ändring i sitt värde vid $t = 0$ ($\mathrm{u}(t)$ är enhetsstegfunktionen). Kvantiteterna u_x och i_x är okända.

Del C. Stationärväxelström

5) [6p]



a) [2p] Skissa ett Bode amplituddiagram (d.v.s. dB mot $\log(\omega)$) för denna nätverksfunktion:

$$H(\omega) = k\frac{(1 + \mathrm{j}\omega/\omega_3)}{(1 + \mathrm{j}\omega/\omega_1)\ (1 + \mathrm{j}\omega/\omega_2)},$$

där k, ω_1, ω_2 och ω_3 alla är positiva reella konstanter. Anta $\omega_1 \ll \omega_2 \ll \omega_3$. Vilket vanligt namn för filtrar (HP,LP,BP,BS) beskriver bäst diagrammet?

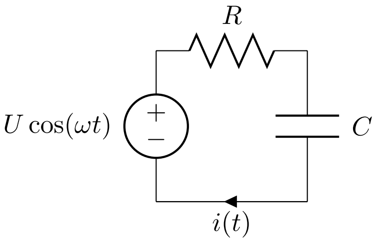
b) [3p] För kretsen till höger, bestäm nätverksfunktionen $H(\omega) = u_{\mathrm{o}}(\omega)/u_{\mathrm{i}}(\omega)$.

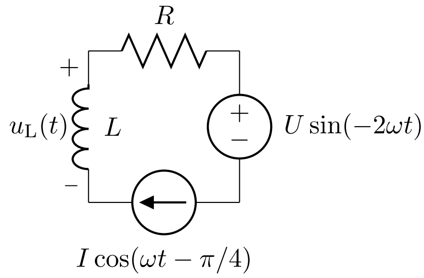
c) [1p] Visa att lösningen $H(\omega)$ från delsvar 'b)' kan uttryckas i samma form som funktionen i delsvar 'a)': bestäm konstanterna k, ω_1, ω_2 och ω_3 som funktioner av kretskomponentvärdena.

6) [6p]

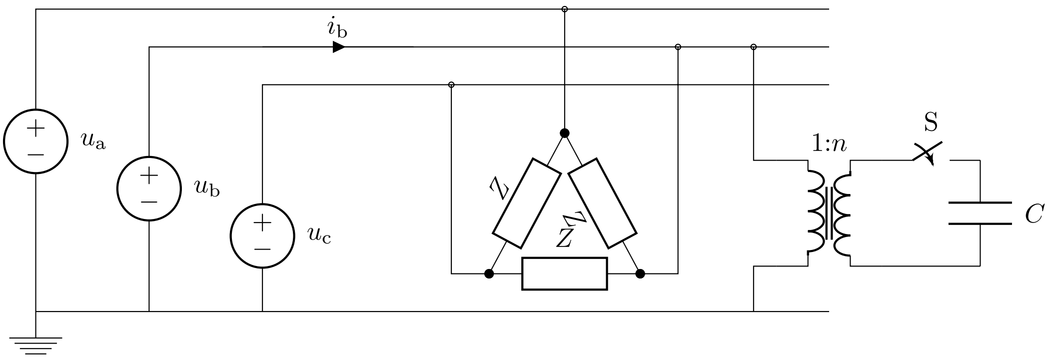
a) [4p] Bestäm $i(t)$ i den *vänstra* kretsen som visas här intill.

b) [2p] Bestäm $u_{\mathrm{L}}(t)$ i den *högra* kretsen.





7) [6p]



Diagrammet ovan visar en ideal trefas spänningskälla vilken försörjer en symmetrisk last och en asymmetrisk last. Källan har vinkelfrekvensen ω; absolutbeloppet av fasspänningen är U_{p} (effektivvärde), och fasföljden är *abc*. Vi räknar helt i frekvensdomänen, med u_{a} som referensfas (0°), och i effektivvärdeskalan. Därför är till exempel visaren för spänningen mellan fas-c och noll-ledaren $u_{\mathrm{c}}(\omega) = U_{\mathrm{p}}\angle{-4\pi/3}$ (eller, ekvivalent, $U_{\mathrm{p}}\angle{2\pi/3}$).

Den symmetriska lasten har Δ-kopplade fasimpedanser $Z = R + \mathrm{j}\omega L$. Den asymmetriska lasten är en kondensator C, vilken kan kopplas genom en brytare till en lindning av en ideal transformator med faktor 1:n. Transformatorns andra lindning är ansluten mellan fas-b och noll-ledaren. (Punktnotationen för lindningarnas riktningar används inte i detta tal eftersom den inte är relevant för svaret.)

Fall 1: brytare 'S' öppet (kondensatorn *ej* ansluten).
Därför är det bara källan och den balanserade lasten som måste betraktas.

a) [1p] Vad är absolutbeloppet av huvudspänningen (d.v.s. spänningen över varje impedans Z)? Du behöver inte visa härledningen av svaret.

b) [2p] Vilken komplexeffekt levereras av källan (hela 'trefas' källan)?

Fall 2: brytare 'S' är nu påslagen (kondensatorn är ansluten) och kvoten n av transformatorn är anpassad för att få perfekt effektfaktorkompensering av strömmen i_{b} som dras från fas-b av källan. Observera att detta gäller bara fas-b: transformatorn med kondensatorn motsvarar en enfas (asymmetrisk) last.

c) [1p] Vad är nu den totala komplexa effekten som levereras av källan?

d) [1p] Vad är strömmen i_{b} (som en visare, inte bara absolutbeloppet)?

e) [1p] Bestäm värdet av n som ger perfekt effektfaktorkompensering i fas-b (som beskriven ovan)? Uttryck n som funktion av kända kvantiteter.

Snipp snapp snut, nu är tentan slut. Glöm ej att använda återstående tid för att dubbelkolla svaren!

Solutions

Here follow quick text-based solutions, including some comments (Nathaniel) about alternative methods. Handwritten solutions are also available in good clear writing (thanks to Xiaolei Wang) and are provided after these ones.

Q1)

The parallel combination of R and $R/2$ has resistance $R/3$. The source I and its parallel resistor $2R/3$ can be source-transformed to the $2R/3$ resistor with a *series* voltage source of $2RI/3$ which (inserting $I = 3U/(4R)$) is $U/2$. This forms a single loop: the total source-voltage (clockwise) is $3U/2$; the total resistance is $R/3+2R/3$ which is R. Thus the current, clockwise, is $3U/(2R)$. By current-division between R and $R/2$, the current i_x is $-2/3$ of the total current (negative because i_x is defined in the anticlockwise direction in the circuit). Hence $i_x = (-2/3)3U/(2R) = -U/R$. What a nice artificial problem with a clean solution!

Q2)

This has strong resemblance to the car-battery problem in the kontrollskrivning.
a) One easy way to find the Thevenin voltage is to use KVL in the loop of U_1,R_1,U_2,R_2. The current clockwise in this loop is $I = (U_1 - U_2)/(R_1 + R_2)$. The voltage between nodes a and b is thus $IR_2 + U_2$, which simplifies to $(U_1R_2 +U_2R_1)/(R_1 +R_2)$. Another way is node analysis, taking just the top node (call the potential V) and using the bottom node as a reference (0): $(V -U_1)/R_1 +(V -U_2)/R_2 = 0$, from which the same result emerges. Hence the Thevenin voltage is $U_\mathrm{T} = (U_1R_2 + U_2R_1)/(R_1 + R_2)$. The Thevenin resistance is (by setting the voltage sources to zero) the parallel combination of R_1 and R_2: $R_\mathrm{T} = R_1R_2/(R_1 + R_2)$.
b) Now that we have the Thevenin equivalent, we can connect it to R_e and find the resulting voltage. By voltage division this is $U_\mathrm{ab} = U_\mathrm{T}R_\mathrm{e}/(R_\mathrm{e} + R_\mathrm{T})$. Expressed in known variables, $U_\mathrm{ab} = \frac{R_\mathrm{e}(U_1R_2+U_2R_1)}{R_1R_2+R_1R_\mathrm{e}+R_2R_\mathrm{e}}$.
c) The current i_x is directly determined because the resistor R_x is in parallel with an independent voltage-source. This current determines the strength of the dependent current source. Hence the dependent current source must have strength $-kU/R_x$. This current source is in series with the aforementioned voltage source and resistor, so their entire combination behaves (externally) like a current source. The only other component is the parallel resistor R_o. Hence the Norton equivalent is simply a current source of kU/R_x pointing from node c to node d, and a resistor R_o in parallel with this. (Alternatively, keep the negative sign and reverse the arrow of the source.)

Q3)

From time $t = 0$, the series combination of R and C has a fixed voltage of U_f applied across it. The time-constant of changes in the period $t > 0$ is thus $\tau = CR$. The equilibrium state (after a long time) of u_c is U_f. The initial state is $u_\mathrm{c}(0) = U_\mathrm{i}$, from the equilibrium before the switch was closed. Knowing that this is a first-order circuit we can immediately write that $u_\mathrm{c}(t) = u_\mathrm{i} + (u_\mathrm{f} - u_\mathrm{i})(1 - \mathrm{e}^{-t/CR})$, or simplify to $u_\mathrm{c}(t) = u_\mathrm{f} + (u_\mathrm{i} - u_\mathrm{f})\mathrm{e}^{-t/CR}$. (So the subscripts f and i meant "final" and "initial"!) More general methods may directly express the ODE (inhomogenous, constant coefficients and forcing) and solve it with the initial condition. We can and should check the answer at least for reasonable values when $t = 0$ and when $t \to \infty$.

Q4)

a) Before $t = 0$ there is equilibrium: constant voltages and currents, and hence no currents in capacitors and no voltage across inductors. Capacitor C_x is open-circuit: its voltage is the voltage across R_5, which by division (with minus sign due to the definition of u_x is $u_x = UR_5/(R_4 + R_5)$. Zero current in C_1 means that the current source's current (equal to I at $t < 0$) must go entirely in the loop R_2 and R_1. Therefore $i_x = I$.
b) Now we are exactly after the step-change in current-source current. There is no change in u_x (it is totally 'isolated' from the influence of the current source by a parallel voltage source, and even if the voltage source were not there, the inductor L_1 capacitor would prevent immediate changes in the rightmost two branches of the circuit, *and* the capacitor C_x would not be able to change its voltage instantaneously); so $u_x = UR_5/(R_4 + R_5)$ as before. The current i_x is tricky to calculate! U is constant, and the voltage on the capacitor C_1 is continuous between $t = 0^-$ and $t = 0^+$. The two thus behave like a voltage source, which must have a value of IR_2 since we know that at $t < 0$ there was no current in R_3. We can draw a circuit with the current source of $2I$ (we can ignore resistor R_1, in series with the source) and its current passing through the parallel combination of R_2 with a series branch of R_3 and voltage source IR_2, then calculate the current i_x in R_2. Let the potential at the top of R_2 relative to the bottom be V: then $(V -R_2I)/R_3 +V/R_2 = 2I$, so $V(1/R_2 +1/R_3) = I(R_2 +2R_3)/R_3$; since $i_x = V/R_2$, we have $i_x = I(2R_3 + R_2)/(R_2 + R_3)$, so $i_x = I(1 + R_3/(R_2 + R_3))$. Alternatively, use of superposition would show us that the current due to the *extra* current I after time $t = 0$ contributes to i_x by current division in R_2 and R_3; we then need only add to this the initial current of I that we calculated for $t < 0$: this gives the above solution.

Q5)

a) We see a gain k, two poles (ω_1 and ω_2) and one zero (ω_3). With the given relations of pole and zero frequencies, we have a constant gain of $20\log_{10}k\,\mathrm{dB}$ at frequencies below ω_1; then the gain falls off at $-20\,\mathrm{dB/decade}$ (i.e. $1/\omega$) until ω_2 at which the gain falls off at $-40\,\mathrm{dB/decade}$ (i.e. $1/\omega^2$) until ω_3, beyond which the fall-off becomes again just $-20\,\mathrm{dB/decade}$. See the handwritten solutions for a sketch. This function is a form of low-pass (LP) filter: flat response at low frequencies, then gain reducing at higher frequencies.
b) Define the opamp's output potential as v_a. The opamp has negative feedback: we assume its inverting and non-inverting inputs have equal potential. Then, by voltage-division in the feedback circuit, we see that $u_\mathrm{i}/v_\mathrm{a} = 1/(1+(R_\mathrm{f}/R_\mathrm{g})/(1+\mathrm{j}\omega C_\mathrm{f}R_\mathrm{f}))$. Inverting and simplifying, $v_\mathrm{a}/u_\mathrm{i} = \frac{R_\mathrm{f}+R_\mathrm{g}}{R_\mathrm{g}} \cdot \frac{1+\mathrm{j}\omega C_\mathrm{f}R_\mathrm{f}R_\mathrm{g}/(R_\mathrm{f}+R_\mathrm{g})}{1+\mathrm{j}\omega C_\mathrm{f}R_\mathrm{f}}$.
The second part of the circuit is independent, as the current it draws from the opamp's output does not affect the feedback loop (the opamp supplies whatever current is needed). Its network function is found by voltage division in L and R_o to be $u_\mathrm{o}/v_\mathrm{a} = 1/(1+\mathrm{j}\omega L/R_\mathrm{o})$. The total network function is therefore the product of the two above parts, $u_\mathrm{o}/u_\mathrm{i} = \frac{R_\mathrm{f}+R_\mathrm{g}}{R_\mathrm{g}} \cdot \frac{1+\mathrm{j}\omega C_\mathrm{f}R_\mathrm{f}R_\mathrm{g}/(R_\mathrm{f}+R_\mathrm{g})}{(1+\mathrm{j}\omega C_\mathrm{f}R_\mathrm{f})(1+\mathrm{j}\omega L/R_\mathrm{o})}$.
c) The result in part 'b)' has already been written in the same form as the function in part 'a)': the corresponding coefficients are therefore obvious by comparison (although ω_1 and ω_2 are obviously interchangeable – you decide which one you want to be due to the LR divider, and which to the opamp feedback!).

Q6)

a) Let us use "the jω method", choosing cosine reference and peak values. Then the voltage source is $U\underline{/0}$, and the current is $i(\omega) = U/Z = U/(R-\mathrm{j}/(\omega C))$. Thus$i(\omega) = (U/\sqrt{R^2+1/(\omega C)^2})\underline{/\tan^{-1}(1/(\omega RC))}$, and $i(t) = \frac{U}{\sqrt{R^2+1/(\omega C)^2}}\cos(\omega t + \tan^{-1}(1/(\omega RC)))$.
b) In other cases, e.g. if the current source were a voltage source, the different frequencies would require use of superposition for a jω solution. However, here the current source is in a single loop, so it entirely determines the loop current. We only need to know this current in order to calculate u_L. We can say "by inspection" that the voltage amplitude will be related to the current amplitude by the factor of reactance, ωL, and phase-shifted by 90° lead: hence $u_\mathrm{L}(t) = -I\omega L\cos(\omega t + \pi/4)$. In the preferred "standard form" with positive real amplitude we would move the negative sign into the argument (180° shift), to give $u_\mathrm{L}(t) = I\omega L\cos(\omega t - 3\pi/4)$. (We'll give the same marks for either form.)

Q7)

a) Line voltage magnitude is $\sqrt{3}U_\mathrm{p}$.

b) Total complex power is three phase-powers, $S_\mathrm{t} = 3\left(\sqrt{3}U_\mathrm{p}\right)^2/Z^* = 9U_\mathrm{p}^2/Z^*$, and we can (if we want) write it out in full (useful in later answers), writing the complex parts on the top, as $S_\mathrm{t} = \frac{9U_\mathrm{p}^2}{R^2+\omega^2L^2}\left(R+\mathrm{j}\omega L\right)$.

c) Total with one phase 'power-factor corrected' is just the old total minus the reactive power of one phase; i.e. multiply the reactive power value (imaginary component) in part b) by 2/3, and leave the active power (real component) the same. $S_\mathrm{t} = \frac{9U_\mathrm{p}^2}{R^2+\omega^2L^2}\left(R+\mathrm{j}\frac{2}{3}\omega L\right)$.

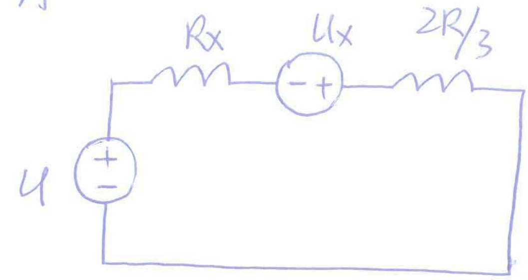
d) Phase b has perfect power-factor correction, so the current is purely resistive. We therefore expect it to have the same phase-angle as the voltage in phase-b. We know the current provides just the active power of *one* phase of the balanced load, i.e. $P_\mathrm{b} = \frac{3U_\mathrm{p}^2R}{R^2+\omega^2L^2}$. The current is thus $i_\mathrm{b} = \left(\frac{3U_\mathrm{p}^2R}{R^2+\omega^2L^2}/U_\mathrm{p}\underline{/-2\pi/3}\right)^*$, so $i_\mathrm{b} = \frac{3U_\mathrm{p}R}{R^2+\omega^2L^2}\underline{/-2\pi/3}$.

e) The reactive power of each phase was $\frac{3U_\mathrm{p}^2\omega L}{R^2+\omega^2L^2}$. Now we're told that for phase b this reactive power demand (due to the balanced load) is to be entirely 'provided' (cancelled) by the capacitor's reactive power. The transformer is ideal, so has no consumption or generation of active or reactive power. The voltage applied to the capacitor is nU_p, so the capacitor *provides* reactive power (i.e. consumes negative reactive power) of $\omega Cn^2U_\mathrm{p}^2$. Equating the two expressions, we get $n = \sqrt{\frac{3L}{C(R^2+\omega^2L^2)}}$ (and would want to choose a positive value of the square-root!).

Woops. I forgot to set a part-question about "dimension analysis". Can't include everything!

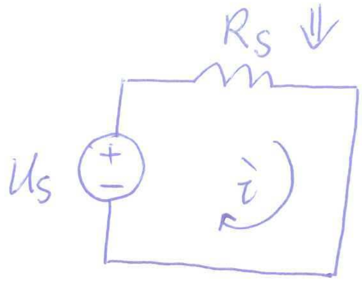
part A

Q 1)



$$R_x = R \parallel \left(\frac{R}{2}\right) = \frac{R \cdot \frac{R}{2}}{R + \frac{R}{2}} = \frac{R}{3}$$

$$U_x = \frac{2R}{3} \cdot I = \frac{2R}{3} \cdot \left(\frac{3U}{4R}\right) = \frac{U}{2} \quad \text{(source transformation)}$$

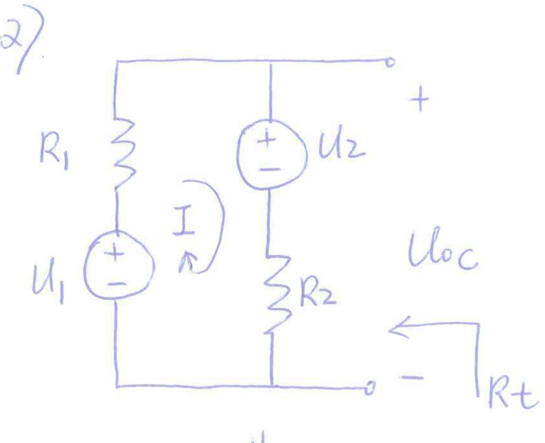


$$i = \frac{U_s}{R_s} = \frac{U + \frac{U}{2}}{\frac{2R}{3} + \frac{R}{3}} = \frac{3U}{2R}$$

So $$i_x = -\frac{R}{\frac{R}{2} + R} \cdot i = -\frac{R}{\frac{R}{2} + R} \cdot \frac{3U}{2R}$$

$$= -\frac{2}{3} \cdot \frac{3U}{2R} = -\frac{U}{R}$$

Q 2)



ⓐ $R_t = R_1 \parallel R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$

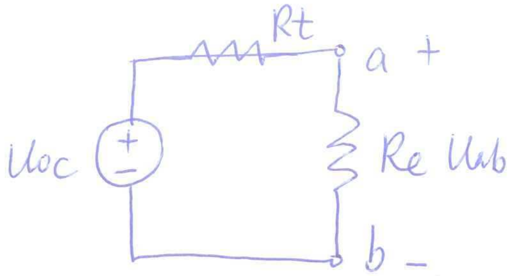
$I R_1 + U_2 + R_2 \cdot I - U_1 = 0$

$\Rightarrow I = \dfrac{U_1 - U_2}{R_1 + R_2}$

$\Rightarrow U_{oc} = U_2 + R_2 \cdot I$

$= U_2 + R_2 \cdot \dfrac{U_1 - U_2}{R_1 + R_2}$

$= \dfrac{R_1 U_2 + R_2 U_1}{R_1 + R_2}$

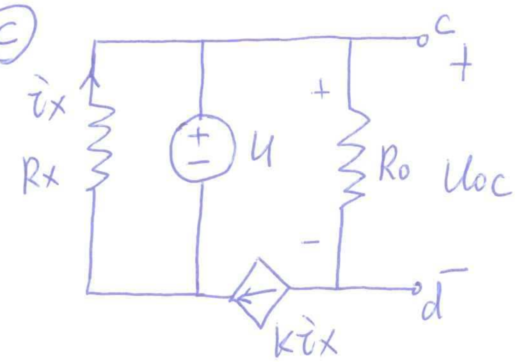


Thevenin equivalent.

ⓑ $U_{ab} = \dfrac{R_e}{R_e + R_t} \cdot U_{oc}$

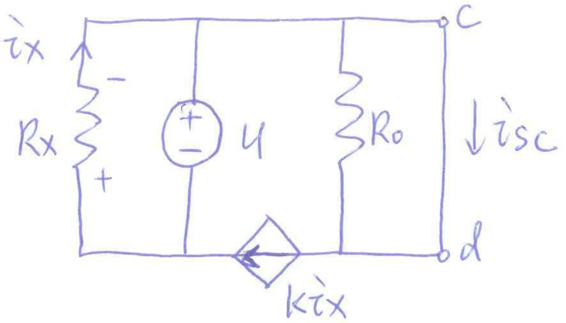
$= \dfrac{R_e}{R_e + \frac{R_1 R_2}{R_1 + R_2}} \cdot \dfrac{R_1 U_2 + R_2 U_1}{R_1 + R_2} = \dfrac{R_e (R_1 U_2 + R_2 U_1)}{R_e (R_1 + R_2) + R_1 R_2}$

ⓒ



$U_{oc} = R_o \cdot k i_x$

$U = -R_x \cdot i_x \Rightarrow i_x = -\dfrac{U}{R_x}$

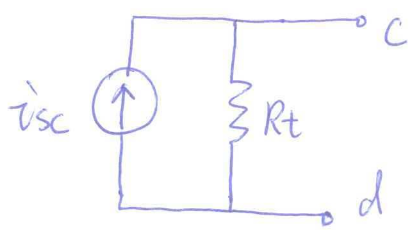


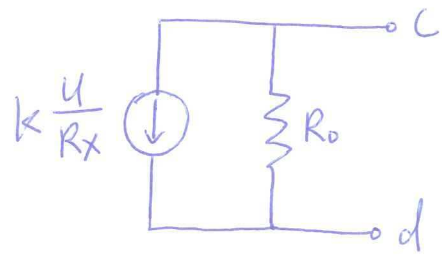
$i_{sc} = k i_x = k \cdot \left(-\dfrac{U}{R_x}\right)$

(because Ro is short circuit)

$\Rightarrow R_t = \dfrac{U_{oc}}{i_{sc}} = R_o$

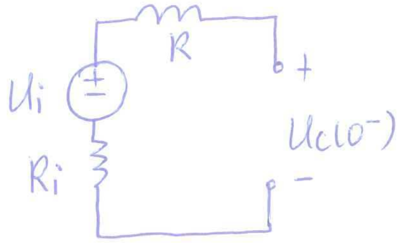
Norton equivalent or





Part B

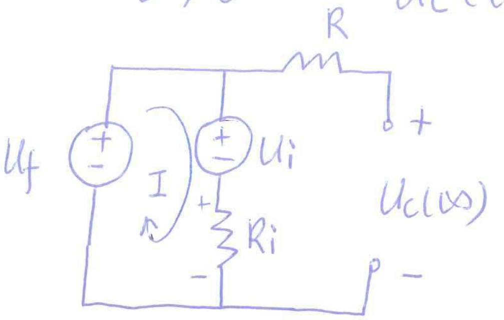
Q3) $t < 0$



$$U_c(0^-) = U_i$$

$$U_c(0^+) = U_c(0^-) = U_i$$

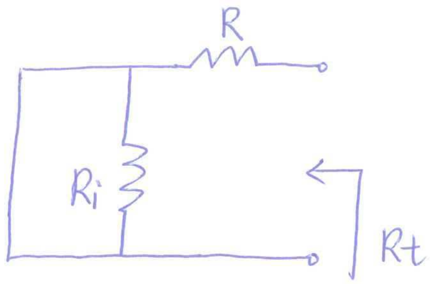
$t > 0$ $\quad U_c(t) = A + B \cdot e^{-t/\tau}$



$$U_c(\infty) = U_f$$

or $\quad I = \frac{U_f - U_i}{R_i}$

$$U_c(\infty) = U_i + R_i \cdot I = U_i + R_i \cdot \frac{U_f - U_i}{R_i} = U_f$$



$$R_t = R$$

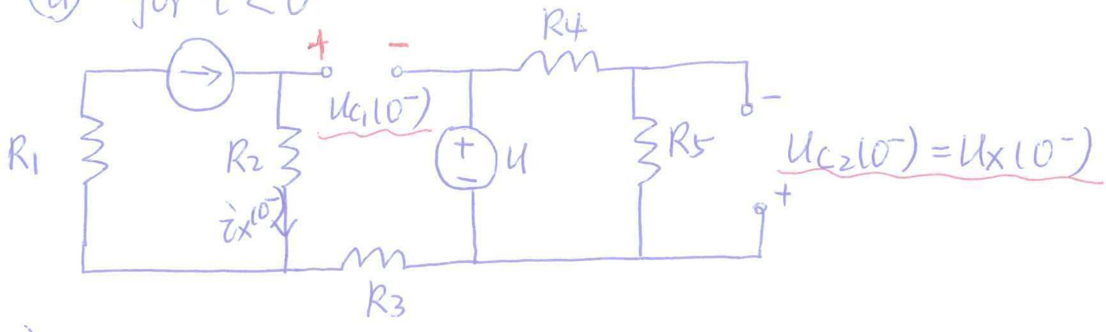
$$\Rightarrow \tau = R_t \cdot C = R \cdot C$$

$$\Rightarrow \begin{cases} U_c(0^+) = A + B = U_i \\ U_c(\infty) = A = U_f \end{cases}$$

$$\Rightarrow A = U_f \quad B = U_i - U_f$$

finally. $\quad U_c(t) = U_f + (U_i - U_f) \cdot e^{-t/RC}$

Q4) ⓐ for $t<0$

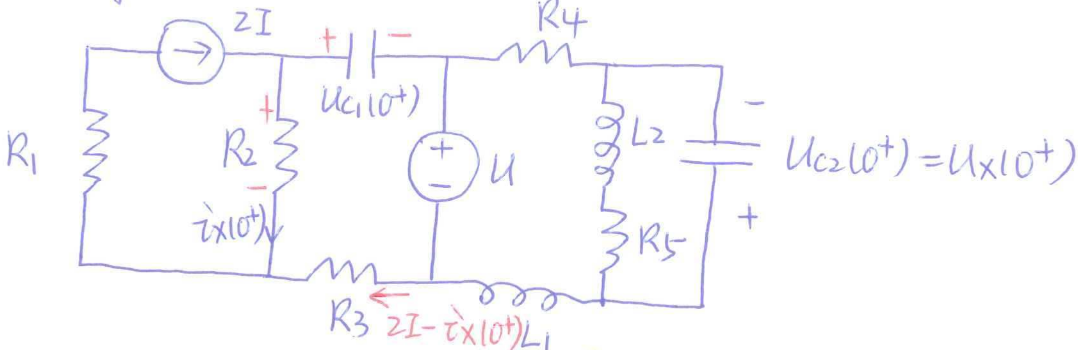


$$i_x(0^-) = I$$

$$u_x(0^-) = u_{c_2}(0^-) = -\frac{R_5}{R_4+R_5}u$$

$$u_{c_1}(0^-) = R_2 \cdot i_x - u = R_2 \cdot I - u$$

ⓑ for $t>0$



$$u_x(0^+) = u_{c_2}(0^+) = -\frac{R_5}{R_4+R_5}u$$

KVL $\quad -R_2 \cdot i_x(0^+) + u_{c_1}(0^+) + u + R_3 \cdot (2I - i_x(0^+)) = 0$

$$u_{c_1}(0^+) = u_{c_1}(0^-) = R_2 I - u$$

$$\Rightarrow -R_2 \cdot i_x(0^+) + R_2 I - u + u + R_3 \cdot (2I - i_x(0^+)) = 0$$

$$(R_2 + R_3) \cdot i_x(0^+) = (R_2 + 2R_3) I$$

$$\Rightarrow i_x(0^+) = \frac{R_2 + 2R_3}{R_2 + R_3} I$$

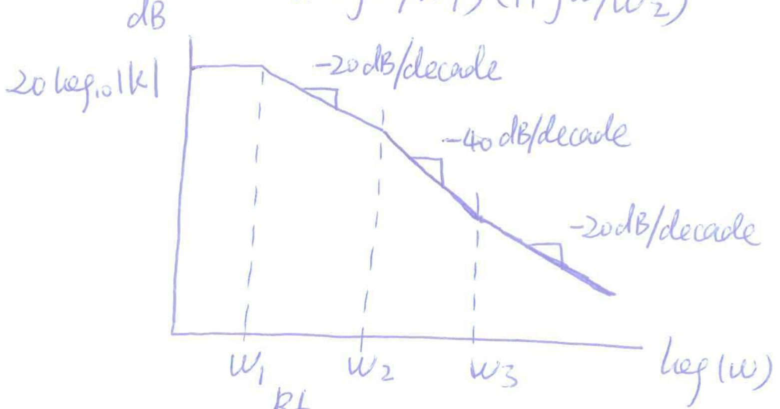
Part C

Q57 ⓐ $H(w) = k \cdot \dfrac{1 + jw/w_3}{(1 + jw/w_1)(1 + jw/w_2)}$

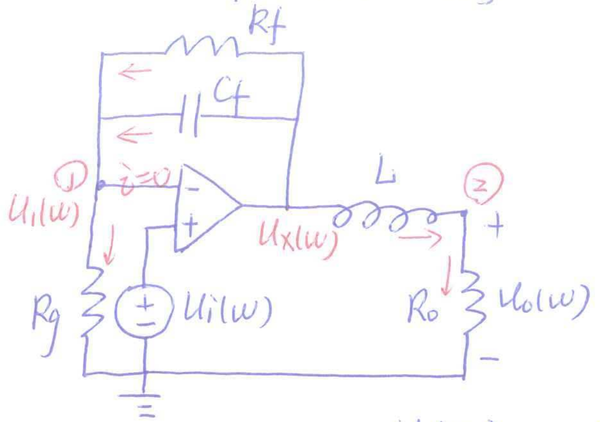
$w_1, w_2 \rightarrow$ poles

$w_3 \rightarrow$ zero

low-pass filter!



ⓑ



KCL at node ① $\dfrac{U_1(w)}{R_g} = \dfrac{U_x(w) - U_i(w)}{R_f \,//\, (\frac{1}{jwC_f})}$

$\Rightarrow \dfrac{U_x(w)}{U_i(w)} = \dfrac{R_f + R_g(1 + jwC_fR_f)}{R_g(1 + jwC_fR_f)} = \dfrac{R_f + R_g}{R_g} \cdot \dfrac{1 + \frac{jwC_fR_fR_g}{R_f + R_g}}{1 + jwC_fR_f}$

KCL at node ② $\dfrac{U_x(w) - U_o(w)}{jwL} = \dfrac{U_o(w)}{R_o}$

$\Rightarrow \dfrac{U_x(w)}{U_o(w)} = \dfrac{R_o + jwL}{R_o} \Rightarrow \dfrac{U_o(w)}{U_x(w)} = \dfrac{R_o}{R_o + jwL} = \dfrac{1}{1 + \frac{jwL}{R_o}}$

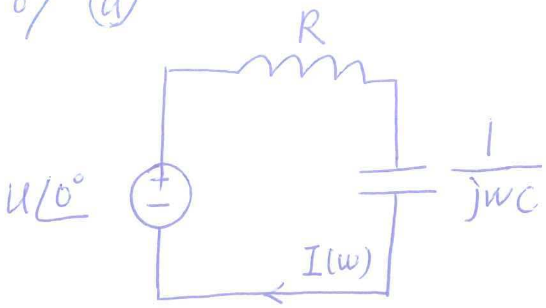
So $H(w) = \dfrac{U_o(w)}{U_i(w)} = \boxed{\dfrac{R_f + R_g}{R_g} \cdot \dfrac{1 + \frac{jwC_fR_fR_g}{R_f + R_g}}{(1 + jwC_fR_f)(1 + \frac{jwL}{R_o})}}$

ⓒ $k = \dfrac{R_f + R_g}{R_g}$

$w_1 = \dfrac{1}{C_fR_f}$ $\quad w_2 = \dfrac{R_o}{L}$ $\quad$ (or $w_1 = \dfrac{R_o}{L}$ $\quad w_2 = \dfrac{1}{C_fR_f}$)

$w_3 = \dfrac{R_f + R_g}{C_fR_fR_g}$

Q 6) (a)



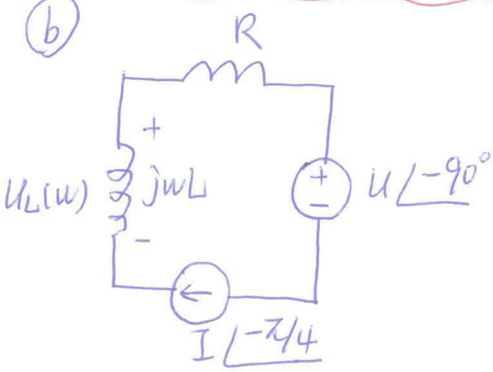
$$I = \frac{U\angle 0°}{R + \frac{1}{j\omega C}} = \frac{U}{R - j\frac{1}{\omega C}}$$

$$|I| = \frac{U}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \qquad \text{angle} = 0 - \arctan(-\frac{1}{\omega CR})$$

$$\Rightarrow I = \frac{U}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \angle \arctan(\frac{1}{\omega CR})$$

$$\Rightarrow i(t) = \frac{U}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos(\omega t + \arctan(\frac{1}{\omega CR}))$$

(b)



$$U_L(\omega) = -j\omega L \cdot I\angle -\pi/4 = -\omega L I \angle -\frac{\pi}{4} + \frac{\pi}{2} = -\omega L I \angle \frac{\pi}{4}$$

$$\Rightarrow U_L(t) = -\omega L I \cos(\omega t + \frac{\pi}{4})$$

Q 7) ⓐ the line voltage magnitude $U_L = \sqrt{3} U_p$

ⓑ the total complex power in three phases

$$S_t = 3U_L I^* = 3U_L \left(\frac{U_L}{Z}\right)^* = \frac{3U_L^2}{Z^*} = \frac{3(\sqrt{3}U_p)^2}{(R+j\omega L)^*}$$

$$= \frac{9U_p^2}{R-j\omega L} = \frac{9U_p^2}{R^2+\omega^2L^2}(R+j\omega L) = P+jQ$$

ⓒ "perfect power-factor correction" means no reactive power in phase b, then the complex power S_t' is

$$S_t' = P + j\frac{2}{3}Q = \frac{9U_p^2}{R^2+\omega^2L^2}\left(R+j\frac{2}{3}\omega L\right)$$

ⓓ the current in phase b is purely resistive, it only provides the real power to the load

$$\frac{1}{3}P = U_p \cdot |I_b| \Rightarrow |I_b| = \frac{3U_pR}{R^2+\omega^2L^2}$$

the angle of phase b is $-2\pi/3$, therefore

$$I_b = \frac{3U_pR}{R^2+\omega^2L^2}\angle -\frac{2\pi}{3}$$

ⓔ the reactive power of each phase is

$$\frac{1}{3}Q = \frac{3U_p^2\omega L}{R^2+\omega^2L^2}$$

which should be cancelled by the capacitor's reactive power Q_t

$$jQ_t = \frac{(nU_p)^2}{\frac{1}{j\omega C}} \Rightarrow Q_t = \omega C n^2 U_p^2$$

Equating two expressions to get

$$n = \sqrt{\frac{3L}{C(R^2+\omega^2L^2)}}$$