KS: EI1120 Elkretsanalys (CENMI), 2014-02-07 kl 10–12

Hjälpmedel: Ett A4-ark med godtyckligt innehåll (handskriven, datorutskrift, diagram, m.m.).

Kontrollskrivningen har 3 tal, med totalt 12 poäng. Den omfattar del A i kursen, 'Likström', och motsvarar del A i tentamen. Betyget på tentan kommer att inkludera del A genom att ta det högre av betygen från KS1 (den här) och från tentans del A. Del A är godkänt vid ≥40%, men glöm inte att tentan kräver 50% räknat över alla delarna.
Var tydlig med diagram och definitioner. Lösningar ska uttryckas i kända kvantiteter, och förenklas.
Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.
Använd återstående tid för att kolla på svaren!

Examinator: Nathaniel Taylor

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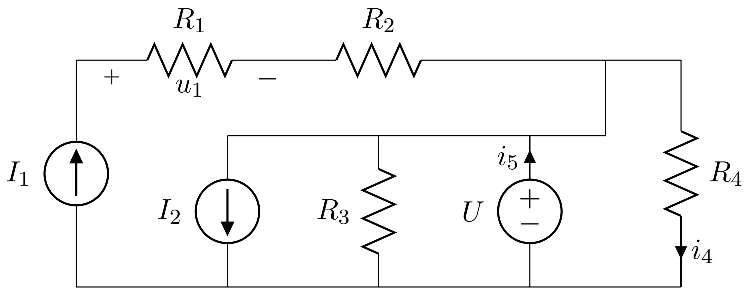
1) [4p]

Känd: I_1, I_2, U, R_1, R_2, R_3, R_4.

Bestäm markerade kvantiteterna:
u_1, i_4, i_5.
Bestäm den totala effekten som *levereras* av *båda* strömkällorna.



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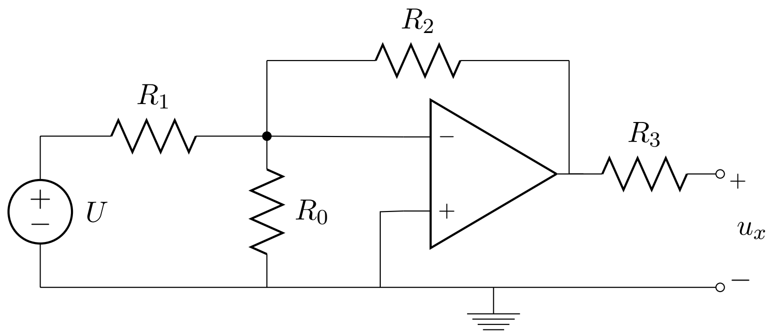
2) [4p]

Känd: R_0, R_1, R_2, R_3, U.
Operationsförstärkaren antas vara idéal.

a) [3p] Bestäm u_x.

b) [1p] Bestäm Thevenin resistansen mellan samma poler som u_x är definierad på.



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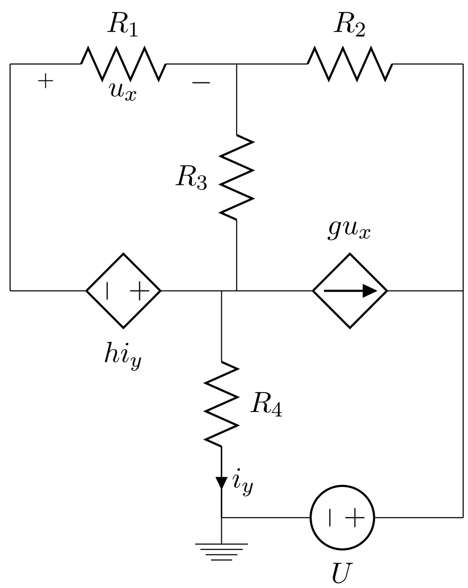
3) [4p]

Känd: R_1, R_2, R_3, R_4, U, g, h.
Identifiera noderna, och definiera en potential till varje. Använd nodanalys för att skriva ekvationer som skulle kunna lösas för att få ut alla nodpotentialer som funktioner av de kända kvantiteterna. Med andra ord, borde man kunna använda dessa ekvationer för att lösa alla nodpotentialer utan att känna till diagrammet.
Du *måste inte lösa* ekvationerna, och *måste inte* skriva om dem i förenklad eller matris form. Men tänk på behovet av lika många oberoende ekvationer som okända variabler. Du kan definiera nya hjälpvariabler, men nya variabler kräver nya ekvationer för att systemet ska vara lösbart.
Det finns flera möjliga svar (alla med samma lösning).

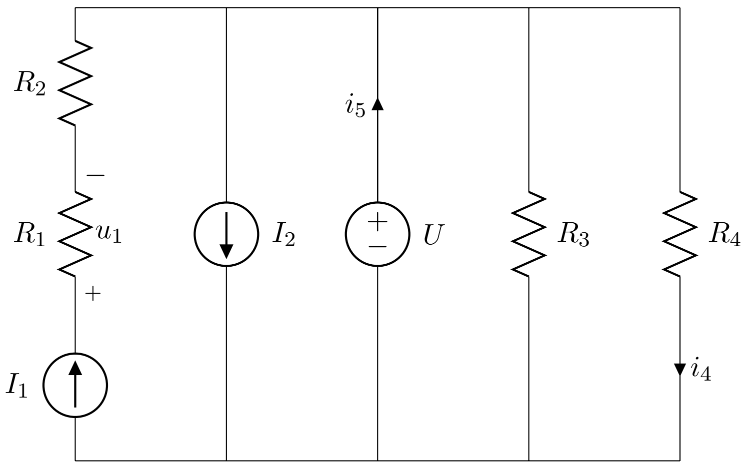


Solutions

For reason of wanting to have solutions ready for the end of the KS *and* wanting to encourage your learning by working out the cause of any mistake, these solutions are very short. Please check against what you remember writing, and consider if you agree. If it's different, try to understand why.
Sometime later I will update with more explanation.

1) $u_1 = I_1 R_1, \qquad i_4 = \frac{U}{R_4}, \qquad i_5 = I_2 + \frac{U}{R_3} + \frac{U}{R_4} - I_1, \qquad P_{\mathrm{I_2+I_2,out}} = I_1((R_1+R_2)I_1 + U) - I_2 U$

The following re-drawing may help in considering these results.



2)

a) $u_x = -\frac{R_2}{R_1} U$

b) $R_{\mathrm{T}} = R_3$

Homework 05 Q1 will help about the analysis of the output voltage, and the irrelevance of R_0.
Homework 05 Q2 will help about the equivalent resistance of this type of circuit.

3) Let's number the nodes 0 (ground), 1 (left), 2 (centre), 3 (top), 4 (right).
You can have made other choices.

Simple way
Define the unknown currents in voltage sources U and hi_y as i_α and i_β respectively, into the sources' + poles (passive).

KCL at all nodes except ground:

$$\mathrm{KCL}(1): \quad 0 = \frac{v_1 - v_3}{R_1} - i_\beta \tag{1}$$

$$\mathrm{KCL}(2): \quad 0 = \frac{v_2 - v_0}{R_4} + i_\beta + g u_x + \frac{v_2 - v_3}{R_3} \tag{2}$$

$$\mathrm{KCL}(3): \quad 0 = \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_4}{R_2} \tag{3}$$

$$\mathrm{KCL}(4): \quad 0 = \frac{v_4 - v_3}{R_2} - g u_x + i_\alpha \tag{4}$$

Now there are 8 unknowns ($v_0 \dots v_4, u_x, i_\alpha, i_\beta$), and 4 equations. If we use the information that one node (v_0) has been defined as a zero-reference (ground), then

$$v_0 = 0 \tag{5}$$

gives a 5th equation. Adding KCL at the ground node does *not* provide a useful equation: with N nodes the Nth node's KCL is just a linear combination of the KCL equations, so it provides no extra information. Instead, use the information given by the voltage sources:

$$\begin{aligned} v_2 - v_1 &= hi_y \quad (6)\\ v_4 - v_0 = v_4 &= U \quad (7) \end{aligned}$$

Now there are 7 equations, but 9 unknowns because i_y has been introduced. So define the controlling variables of the dependent sources in terms of existing variables – then there are 9 equations and 9 unknowns.

$$u_x = v_1 - v_3 \tag{8}$$

$$i_y = \frac{v_2}{R_4} \tag{9}$$

The systematic way in which this was done is important! There are plenty of ways to write a sufficient set of equations, but we cannot just be confident that "n unknowns, n equations, therefore it's all ok" is true. The above method of handling $N-1$ nodes, then ground potential and the information given by voltage-sources, then defining controlling variables in terms of known variables, is one way to develop linearly independent equations.

Supernode

Nodes 0 and 4 become a (ground) supernode; we choose to always use U instead of v_4 in the equations (and 0 instead of v_0).

Nodes 1 and 2 become another supernode; we choose to define unknown potential v_2, and always write $(1 - h/R_4)v_2$ instead of v_1.

Node 3 is a further node, with unkown potential v_3.

We define the dependent current-source's current in terms of our chosen node potentials as $gu_x = g((1 - h/R_4)v_2 - v_3)$.

There are now only two unkown variables: v_2 and v_3.
Writing KCL at the two non-ground nodes/supernodes,

$$0 = \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} + \frac{(1 - h/R_4)v_2 - v_3}{R_1} + g\left(\left(1 - \frac{h}{R_4}\right)v_2 - v_3\right) \tag{10}$$

$$0 = \frac{v_3 - (1 - h/R_4)v_2}{R_1} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - U}{R_2} \tag{11}$$

To do as the question required, one should also write as equations the earlier statements that would let us define v_1 and v_4 after the above equations are solved for v_2 and v_3,

$$v_1 = \left(1 - \frac{h}{R_4}\right)v_2 \tag{12}$$

$$v_4 = U \tag{13}$$

If you want to play with SPICE, Matlab and Mathematica solutions, see the LaTeX source-code (from the webpage); it is a plain text file, including commented sections with these calculations.