# KS2: EI1120 Elkretsanalys (CENMI), 2014-02-20 kl 10–12

**Hjälpmedel:** Ett A4-ark med godtyckligt innehåll (handskriven, datorutskrift, diagram, m.m.).

Kontrollskrivningen har 2 tal, med totalt 10 poäng. Den omfattar del B i kursen, 'Transient', och motsvarar del B i tentamen. Betyget på tentan kommer att inkludera del B genom att ta det högsta av betygen från KS2 (den här) och från tentans del B. Del B är godkänt vid ≥40%, men glöm inte att tentan kräver 50% räknat över alla delarna.

Var tydlig med diagram och definitioner. Lösningar ska uttryckas i kända kvantiteter, och förenklas. Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.
Använd återstående tid för att kontrollera svaren!

Examinator: Nathaniel Taylor

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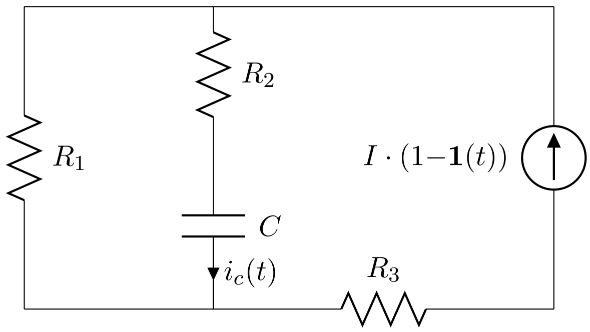
**1)** [5p]

Bestäm strömmen $i_c(t)$ i kondensatorn som tidsfunktion för perioden $t > 0$.

Kretsen kan antas vara i jämviktsläge innan tiden $t = 0$. Observera att enhetsstegfunktionen är skriven här som $\mathbf{1}(t)$. Alla markerade komponentvärden är kända.

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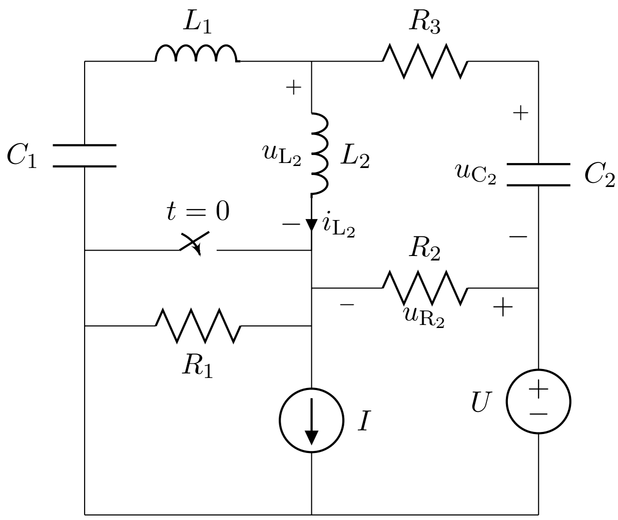
**2)** [5p]

Alla markerade komponentvärden är kända, t.ex. $L_1$, $U$. Alla markerade spänningar och strömmar är okända, t.ex. $u_{R_2}$. Brytaren stänger vid tid $t = 0$.

**a)** [3p] Betrakta jämviktsläget innan brytaren stänger, d.v.s. $t = 0^-$.
Bestäm $i_{L_2}$, $u_{R_2}$ och $u_{C_2}$.

**b)** [2p] Betrakta tiden direkt efter att brytaren stänger, $t = 0^+$.
Bestäm $u_{L_2}$ och $u_{R_2}$.




Tänk på att rita om kretsen med rätt information för fallet som du analyserar. Dubbelkolla omritningen. Var systematisk. Var tydlig med var slutsvaren är!

# Solutions, KS2

**1)**

The current source is $I \cdot (1 - \mathbf{1}(t))$, which means it is $I$ for $t < 0$ and 0 for $t > 0$.

In the equilibrium, the state (continuous variable) of the capacitor is $u_c(0^-)$. The value can be found by setting the capacitor to be an open circuit: then all current $I$ passes through $R_1$, and no current passes through $R_2$, so the voltage across the open-circuited capacitor is $IR_1$.

At $t > 0$ the "zero current source" can be seen as an open circuit, so the branch on the right side can be ignored. The capacitor voltage is initially $u_c(0^+) = u_c(0^-) = IR_1$. The final value of capacitor voltage will be zero, as the capacitor is connected in a loop with $R_1$ and $R_2$. You could reason about why this means it will discharge, or you could just see the two as a Thevenin source with zero voltage, and consider that the equilibrium voltage of a capacitor connected to a Thevenin source is the open-circuit voltage of the source.

We can find the equation for capacitor voltage by the equivalent-circuit method or by writing the equation. It will be

$$u_c(t) = IR_1 \, \mathrm{e}^{-\frac{t}{C(R_1+R_2)}}$$

But the *current* is requested; it has been defined with passive convention relative to the voltage, so we have $i_c(t) = C\frac{\mathrm{d}u_c(t)}{\mathrm{d}t}$. Therefore,

$$i_c(t) = -\frac{CIR_1}{C(R_1+R_2)} \, \mathrm{e}^{-\frac{t}{C(R_1+R_2)}} = -\frac{IR_1}{R_1+R_2} \, \mathrm{e}^{-\frac{t}{C(R_1+R_2)}}.$$

This tells us that the current comes out of the top of the capacitor, first quickly (at $t \simeq 0$, when the capacitor has a voltage $\simeq IR_1$), then slower and slower as the capacitor discharges.

The result could instead have been found by considering initial and final values of the current, or by finding and solving a differential equation for the current.

**2)**

Quick answers, for anyone who wants to check. Please think about any differences, and try to understand why they exist ... one or both of us is presumably wrong....

**a)** Equilibrium, before the switch closes: $t = 0^-$.

* $i_{\mathrm{L}_2} = 0$

Thie is because all paths through $L_2$ are blocked by a series capacitor (open circuit in equilibrium).

* $u_{\mathrm{R}_2} = U - \frac{(U-IR_2)R_1}{R_1+R_2} = \frac{U(R_1+R_2)-(U-IR_2)R_1}{R_1+R_2} = \frac{(U+IR_1)R_2}{R_1+R_2}$

This is from nodal analysis (or other method) on the lower part of the circuit, after identifying that the capacitors block all current in the upper part.

* $u_{\mathrm{C}_2} = -u_{\mathrm{R}_2} = \frac{(-U-IR_1)R_2}{R_1+R_2}$

This is seen from the fact that $C_2$ causes $R_3$ to have no current, and therefore no voltage, and $L_2$ is treated as a short-circuit. There is therefore the same voltage across $C_2$ and $R_2$, but the definitions are in opposite directions.

**b)** Continuity, immediately after the switch closes: $t = 0^+$.

* $u_{\mathrm{L}_2} = U + u_{\mathrm{C}_2}(0^+) = U + u_{\mathrm{C}_2}(0^-) = U - \frac{(U+IR_1)R_2}{R_1+R_2} = \frac{(U-IR_2)R_1}{R_1+R_2}$

There is still no current in $R_3$, as the only path through it passes through inductors ($L_1$ in one way, $L_2$ in the other) which still have zero current at $t = 0^+$. Taking KVL around the loop of the switch, $U$, $C_2$, $R_3$ and $L_2$, we find that $u_{\mathrm{L}_2} = U + u_{\mathrm{C}_2}$, leading to the above.

* $u_{\mathrm{R}_2} = U$

Notice that $R_2$ is now in parallel with the voltage source.