KTH EI1120 Elkretsanalys (CENMI), Tentamen 2014-03-20 kl 08–13

Hjälpmedel: Ett A4-ark med studentens anteckningar (båda sidor).

Svar får anges på svenska eller engelska. En kort ordlista finns på sista sidan.

Läs varje tal noggrant **innan du försöker svara**.
Tänk på att **använda återstående tid till att kolla igenom varje svar**: man kan göra dimensionsanalys, rimlighetsbedömning (t.ex. "är det rätt att y går ner medan x går ner?"), och lösning genom en alternativ metod. Lösningar ska **förenklas** om inte annat är specificerat.
Satsa inte för mycket tid på bara en uppgift om du fastnar: ta hänsyn till poängvärden på uppgifterna. Det är ofta så att **senare** *del*tal är betydligt **svårare** än de första deltalen.

Tentan har 8 tal i 3 delar: 3 i del A (12p), 2 i del B (10p) och 3 i del C (18p).
Räknande av betyg: Låt A, B och C vara de maximala möjliga poängen från delarna A, B och C i tentan, d.v.s. A=12, B=10, C=18. Låt a, b och c vara poängen man får i dessa respektive delar i tentan, och a_{k} vara poängen man fick från kontrollskrivning KS1, och b_{k} poängen från KS2, och h bonuspoängen från hemuppgifterna. Godkänd tentamen (och därigenom hel kurs) kräver:

$$\frac{\max(a, a_{\mathrm{k}})}{A} \geq 0{,}4 \quad \& \quad \frac{\max(b, b_{\mathrm{k}})}{B} \geq 0{,}4 \quad \& \quad \frac{c}{C} \geq 0{,}3 \quad \& \quad \frac{\max(a, a_{\mathrm{k}}) + \max(b, b_{\mathrm{k}}) + c + h}{A + B + C} \geq 0{,}5.$$

Betyget räknas också från summan över alla delar och bonuspoäng, d.v.s. sista termen ovan!
Betygsgränserna (%) är 50 (E), 60 (D), 70 (C), 80 (B), 90 (A). Är betyget mellan 44 och 50%, eller bara en av delarna av tentan underkänd trots bra betyg i de andra, så kan betyget Fx registreras, med möjlighet att få betyget E om ett kompletteringsarbete är godkänt inom några veckor efter tentamen. Se PM:et angående rättningsnormer och överklagande. Instruktionerna ovan tar prioritet över PM vid skillnad.

Examinator: Nathaniel Taylor

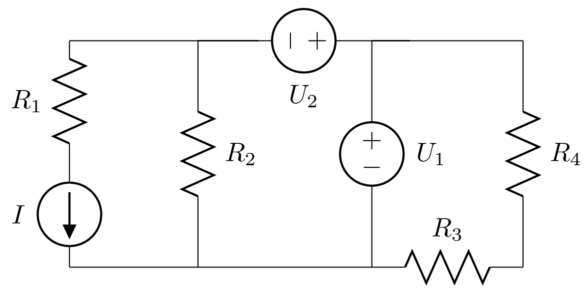
Del A. Likström

1) [4p]
Kända: R_1, R_2, R_3, R_4, I, U_1, U_2.

a) [1p] Bestäm effekten levererat till R_1.

b) [1p] Bestäm effekten levererat till R_3.

c) [2p] Bestäm effekten levererat från källan U_2.

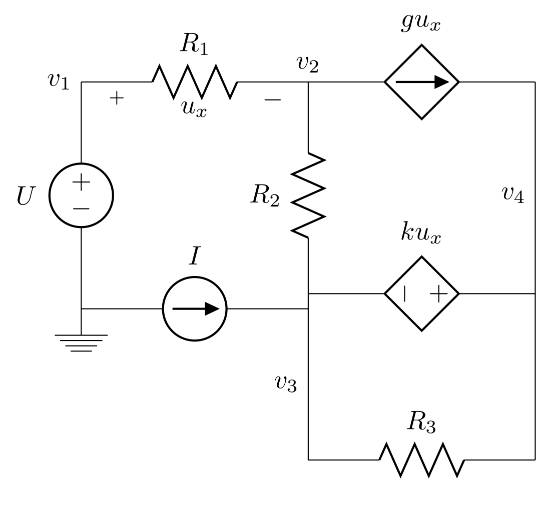


2) [4p]
Kända: R_1, R_2, R_3, U, I, k, g.

Använd nodanalys för att skriva ekvationer som går att lösa för de okända potentialerna v_1, v_2, v_3, v_4.

Du *måste inte lösa* ekvationerna, och *måste inte* skriva om dem i förenklad eller matris form. Du får definiera hjälpvariabler (men ekvationerna måste räcka till att unikt bestämma potentialerna).

Det finns flera möjliga svar (alla med samma lösning). Förmodligen är det bäst att använda ett systematiskt sätt att skriva ekvationerna.

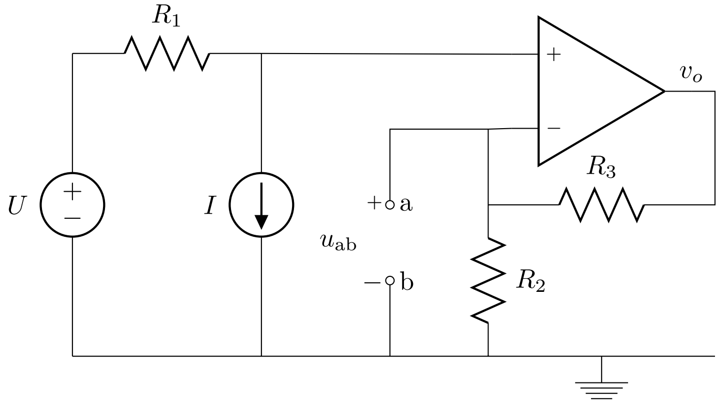


3) [4p]
Kända: U, I, R_1, R_2, R_3.
Operationsförstärkaren antas vara idéal.

a) [2p] Bestäm v_o.

b) [1p] Bestäm u_{ab}.

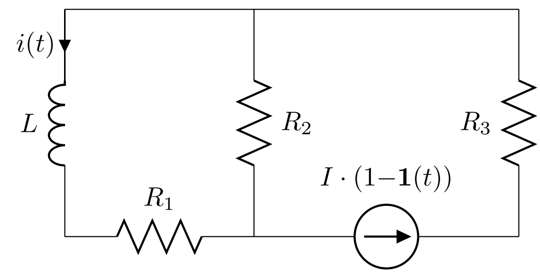
c) [1p] Vad är Theveninekvivalenten mellan polerna a och b? (Svårt: konceptfråga.)



Del B. Transient

4) [5p]
Kända: I, L, R_1, R_2, R_3.
Kretsen är i jämviktsläge innan tiden $t = 0$.
Enhetsstegfunktionen är skriven här som $\mathbf{1}(t)$.

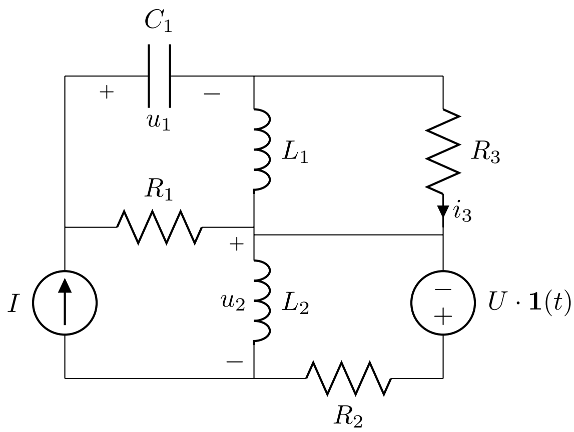
Bestäm strömmen $i(t)$ i spolen, som tidsfunktion för perioden $t > 0$.



5) [5p]
Kända: I, U, R_1, R_2, R_3, L_1, L_2, C_1.
Enhetsstegfunktionen är skriven här som $\mathbf{1}(t)$.

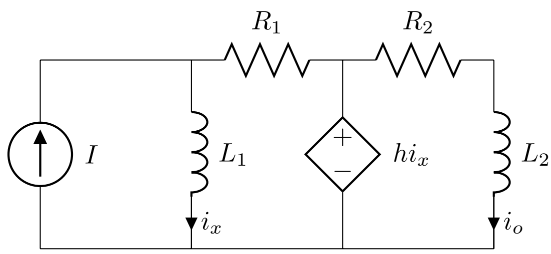
a) [3p] Betrakta jämviktsläget vid $t = 0^-$.
Bestäm $u_1(0^-)$, $u_2(0^-)$ och $i_3(0^-)$.

b) [2p] Betrakta tiden $t = 0^+$.
Bestäm $u_1(0^+)$, $u_2(0^+)$ och $i_3(0^+)$.



Del C. Växelström

6) [6p]
Kända: R_1, L_1, h, R_2, L_2.
I beskriver en växelströmskälla; den behövs inte i svaren, då uppgiften handlar om nätverksfunktioner mellan andra variabler och I.



a) [3p] Skissa ett Bode amplituddiagram av funktionen

$$H(\omega) = \frac{k}{(1 + \mathrm{j}\omega/\omega_1)(1 + \mathrm{j}\omega/\omega_2)}.$$

Anta att $\omega_1 \ll \omega_2$, och att $k > 1$. Markera viktiga punkter och lutningar.

b) [2p] Bestäm nätverksfunktionen $H(\omega) = \frac{i_o(\omega)}{I(\omega)}$ av kretsen ovan.

c) [1p] Visa att svaret till deltal 'b' kan skrivas i samma formen som funktionen från deltal 'a' (med k, ω_1 och ω_2 reella och positiva; antа $R_1 > h$). Det räcker att uttrycka k, ω_1 och ω_2 i kända variabler.

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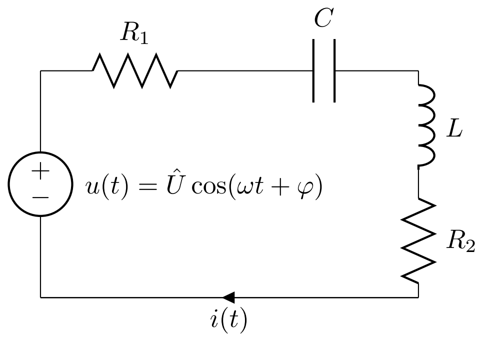
7) [6p]
Kända: $\hat{U}$, ω, φ, R_1, C.
Observera att beräkningarna här kan göras med växelströmsanalys (komplexa tal), men deltal 'c' kräver omvandling till en tidsfunktion.



a) [2p] Bestäm R_2 och L för att maximera effektutvecklingen i R_2.

I alla följande deltal ska R_2 och L ha de värden som bestämdes i deltal 'a'. Har du inte löst deltal 'a', så kan du behålla symbolerna R_2 och L i senare lösningar.

b) [1p] Bestäm den genomsnittliga effekten ("aktiveffekt") i R_2.

c) [2p] Bestäm $i(t)$.

d) [1p] En annan spole, med självinduktans L_2, placeras nära spolen L. Spolen L har fortfarande självinduktans L. Det finns en ömsesidig induktans mellan dessa två spolar: kopplingskoefficienten mellan spolarna är k. Den andra spolen har ingen anslutning till en krets (den är öppenkrets). Bestäm amplituden (storleken) av spänningen över den andra spolen. Värdena L_2 och k kan betraktas som kända.

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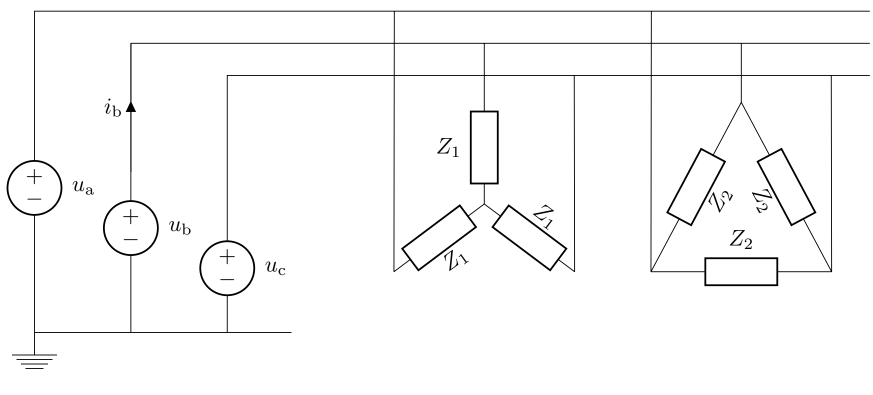
8) [6p]
Kända: U, ω, R, L, C.
Källorna i diagrammet är växelströmskällor med vinkelfrekvens ω och spänning (effektivvärde) U.
De kan beskrivas med fasvektorer $u_\text{a} = U\underline{/0}$, $u_\text{b} = U\underline{/-120°}$ och $u_\text{c} = U\underline{/-240°}$.
Varje impedans Z_1 är en kondensator C, och varje impedans Z_2 är en seriekopplade mostånd R och spole L.



a) [3p] Vilken aktiveffekt försörjs av hela trefaskällan (alla tre spänningskällor)?

b) [2p] Vilket värde måste C ha (uttryckt i ω, R, och L) för att ingen reaktiveffekt dras från källan?

c) [1p] Bestäm i_b (magnitud och fas) när C är valt enligt deltal 'b'.

Ordlista över mindre självklara översättningar: current *ström*, voltage *spänning*, power *effekt*, rms value *effektivvärde*, phasor *fasvektor*, source *källa*, unit-step *enhetssteg*, terminal *pol*, mutual inductance *ömsesidiginduktans*, opamp (operational amplifier) *operationsförstärkare*, angular(radian) frequency *vinkelfrekvens*, equilibrium *jämviktsläge*, inductor (coil) *spole*, active power *aktiveffekt*.

Solutions (EI1120, VT14, 2014-03-20)

These solutions are written mainly for checking the final answers; the homework solutions give longer descriptions of methods that can be used.

Q1

a) R_1 is in series with current source I; therefore the current in R_1 is I.
The power is then $I^2 R_1$. (The direction of current is not important.)

b) The series combination of R_3 and R_4 has a voltage source U_1 connected to it.
The current in R_3 is therefore $\frac{U_1}{R_3+R_4}$, so the power into R_3 is $\left(\frac{U_1}{R_3+R_4}\right)^2 R_3$.
Another possible method uses voltage-division between R_3 and R_4 to give $u_{\mathrm{R3}} = \frac{U_1 R_3}{R_3+R_4}$; then $P = u_{\mathrm{R3}}^2 / R_3$.

c) We know the voltage of U_2, and we need to find the current through it (including direction) in order to calculate the power coming from this source.
Look at the node to the left of U_2: one branch contains source I, pointing downward; the other branch is R_2, in which the current downward is $\frac{U_1-U_2}{R_2}$. By KCL in the top left node, the total current in U_2, using the active convention, is $-I - \frac{U_1}{R_2} + \frac{U_2}{R_2}$.
The power output is therefore $-IU_2 - \frac{U_1 U_2}{R_2} + \frac{U_2^2}{R_2}$.

Q2

By the "simple method" for writing nodal equations:

Define current i_{a} in voltage-source U, and i_{b} in voltage-source ku_x, both with passive convention (going in at + terminal).

Then by KCL at every node except the ground node,

$$
\begin{aligned}
\frac{v_1 - v_2}{R_1} + i_{\mathrm{a}} &= 0 \qquad \mathrm{KCL}(1)_{\mathrm{out}} \\
\frac{v_2 - v_1}{R_1} + \frac{v_2 - v_3}{R_2} + g u_x &= 0 \qquad \mathrm{KCL}(2)_{\mathrm{out}} \\
\frac{v_3 - v_2}{R_2} + \frac{v_3 - v_4}{R_3} - I - i_{\mathrm{b}} &= 0 \qquad \mathrm{KCL}(3)_{\mathrm{out}} \\
\frac{v_4 - v_3}{R_3} - g u_x + i_{\mathrm{b}} &= 0 \qquad \mathrm{KCL}(4)_{\mathrm{out}}
\end{aligned}
$$

then include the potential-differences imposed by the two voltage sources,

$$
\begin{aligned}
v_1 - 0 &= U \\
v_4 - v_3 &= k u_x,
\end{aligned}
$$

and include the definition of the controlling variable u_x in terms of node potentials,

$$
u_x = v_1 - v_2,
$$

noting that in this case there is just one such variable, which controls both of the dependent sources.

It is *not* required that the above be solved and rearranged neatly to show the node potentials. But if you want to, then . . .

$$
\begin{aligned}
v_1 &= U \\
v_2 &= U + I R_1 \\
v_3 &= U + I(R_1 + R_2 - g R_1 R_2) \\
v_4 &= U + I(R_1(1-k) + R_2 - g R_1 R_2)
\end{aligned}
$$

Of course, the supernode method or various in-betweens could have been used instead of the above. If the supernode method is used, then sufficient equations should be given to allow the different potentials inside a supernode to be calculated even by someone who sees only the equations, not the circuit.

The supernode method is good for hand calculation or proofs, but has more potential for error in writing, since it involves doing some simplifications. If there's a computer available, one might as well take advantage of writing the equations in the "simple" form that clearly corresponds to the actual circuit, and that solves for the currents in voltage sources as well as for the node potentials; then one gets more information, and it is easier to modify the equations to match a modified circuit.

Q3

a) The potential at the non-inverting input, v_+, is $v_+ = U - IR_1$.
Due to the assumption of negative feedback and an ideal opamp, the inverting input v_- is also held to this value: $v_- = v_+ = U - IR_1$.
By voltage division, $v_- = \frac{R_2}{R_2+R_3} v_o$.
So $v_o = \frac{(U-IR_1)(R_2+R_3)}{R_2}$.

b) Voltage $u_{\rm ab}$ is the same as potential v_-, as the terminals a and b are connected to the opamp's inverting input and ground.
From part 'a', $v_- = U - IR_1$.
Therefore, $u_{\rm ab} = U - IR_1$.

c) The Thevenin voltage is the open-circuit voltage, which from part 'b' is $U_{\rm T} = u_{\rm ab} = U - IR_1$.
The Thevenin resistance is zero: $R_{\rm T} = 0$.

The resistance won't solve nicely by just trying a short-circuit: that would give a contradiction, by putting a claim that $u_{\rm ab} = 0$ (short-circuit) together with a claim that $u_{\rm ab} = U - IR_1$ (opamp feedback). The reason is that the equivalent source has zero resistance.

It would be possible to solve it by realising that $u_{\rm ab}$ stays the same even if different currents flow between the terminals a-b. This is due to the feedback condition; if we change the current between a-b by connecting resistors or current sources, then it is v_o that has to change, until it has forced $v_- = v_+$ again. That constant voltage represents a horizontal line in the v, i plane, indicating a Thevenin source with zero impedance.

Or one could be more equation-based: define some finite current between a-b (e.g. put a current-source there) and calculate the voltage $u_{\rm ab}$; then calculate the equivalent resistance from this and the open-circuit case.

As was noted in the question, this is a "concept question"; it has low available points, so is not much of a loss; it is hoped to be stimulating to those who think with diagrams.

Q4

In equilibrium, before $t = 0$, the inductor is like a short-circuit, and the current in the inductor can therefore be found by current division between R_1 and R_2, $i(0^-) = \frac{IR_2}{R_1+R_2}$.

An *inductor's current* is a continuous variable, so $i(0^+) = i(0^-) = \frac{IR2}{R_1+R_2}$.
After $t = 0$ the current source has zero current, so is an open circuit: the whole right-hand branch can therefore be ignored. The inductor's current therefore has to flow in the loop of L, R_1, R_2, so the circuit's time-constant is $\tau = \frac{L}{R_1+R_2}$.
The final value is zero, $i(\infty) = 0$.

Hence, by one method or another, $i(t) = I \frac{R_2}{R_1+R_2} \, {\rm e}^{-t \frac{R_1+R_2}{L}}$.

Q5

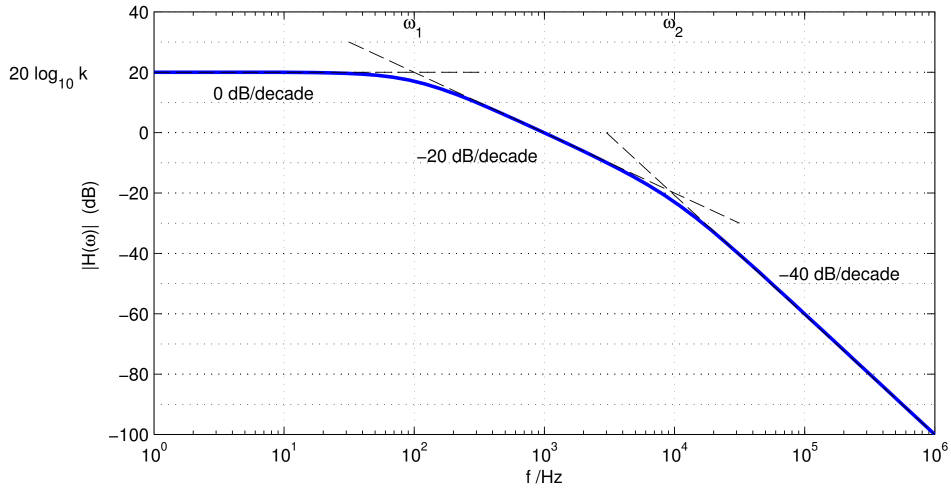
It is extremely helpful to do a complete re-drawing of the diagram for each state that is analysed, with suitable replacements of L and C components. See the notes or homeworks in this topic for examples.

a) This is an equilibrium with constant source-values; all inductors can be replaced by short-circuits, and all capacitors by open-circuits.
$u_1(0^-) = IR_1$ (open-circuit, parallel with R_1; all I has to go through R_1)
$u_2(0^-) = 0$ (L_2 is a short-circuit)
$i_3(0^-) = 0$ (R_3 is in parallel with short-circuited L_1)

b) This is the state immediately after a change in a source value; continuity applies to capacitor voltages and inductor currents between times 0^- and 0^+.
The inductors and capacitors are replaced by respectively current-sources and voltage-sources, having the same values as they had at $t = 0^-$ (note: *only* valid at $t = 0^+$; the values can change after that time).
$u_1(0^+) = IR_1$ (seen directly, by continuity)
$u_2(0^+) = -U$ (fairly hard)
$i_3(0^+) = 0$ (fairly hard)

Q6

a) The numbers in the example plot (below) are arbitrary. The points where the gradient changes should be marked to show ω_1 and ω_2. The gain at low frequency should be $> 0\,\text{dB}$, since we are told that $k > 1$; ideally, you should mark the point $20\log_{10} k$ where your line meets the y-axis.



This plot shows the true amplitude response (smooth curve) done by computer plotting, and some asymptotic lines. You only need to draw three straight lines that give the asymptotic approximation.

b) This circuit could be analysed in several ways. Node analysis is a good choice.

Let us make the bottom node be ground: then the node above the voltage source is part of the ground supernode, and just two nodes remain for the analysis.

Define the top left node as potential v. KCL (out) at this node gives $-I + \frac{v}{\mathrm{j}\omega L_1} + \frac{v - hi_x}{R_1} = 0$.
The controlling variable of the voltage source is i_x, which can be defined as $i_x = \frac{v}{\mathrm{j}\omega L_1}$.
Then the current i_o can be found as $i_o = \frac{hi_x}{R_2 + \mathrm{j}\omega L_2}$, or more "formally" by doing KCL on the top-right node.
Putting the above results together gives,

$$H = \frac{\frac{hR_1}{R_2(R_1-h)}}{(1 + \mathrm{j}\omega L_1/(R_1 - h))\,(1 + \mathrm{j}\omega L_2/R_2)}$$

c) One can choose $\omega_1 = \frac{R_1-h}{L_1}$ and $\omega_2 = \frac{R_2}{L_2}$ (or define the $\omega_{1,2}$ values the other way round), and $k = \frac{hR_1}{R_2(R_1-h)}$.
This way, the network function derived in part 'b' for the shown circuit can be fitted to the function that was plotted in part 'a'.

Note that the relative size of R_2 and h affects the sign of the results, due to the $R_1 - h$ terms; the situation where ω_1, ω_2 and k are positive is when $R_1 > h$.
However, this detail does not affect the *amplitude* response, since it only changes the sign of the whole expression (through k) and the sign of an imaginary part of one pole term (through one of the frequencies $\omega_{1,2}$).

Q7

a) Maximum power implies that the impedance that we can affect $(R_2 + \mathrm{j}\omega L)$ must be the complex conjugate of the source impedance of the rest of the circuit $(R_1 - \mathrm{j}\frac{1}{\omega C})$. So, $R_2 + \mathrm{j}\omega L = R_1 + \mathrm{j}\frac{1}{\omega C}$.
This tells us $R_2 = R_1$ and $L = \frac{1}{\omega^2 C}$.

b) Noting that L and R_2 are now defined in terms of C and R_1 (from part 'a'), we know that the impedances of the capacitor, $\frac{1}{j\omega C}$, and inductor, $j\omega L$, must sum to zero.

So the total impedance around the circuit is just $2R_1$. The active power in R_2 is therefore $P = \frac{\hat{U}}{8R_1}$.
The 8 comes from the total resistance being $2R_1$, and the total power being divided between the 2 equal resistors, and the $\hat{U}$ being a peak (not rms) value: $2^3 = 8$.

c) The current in the loop is $i(\omega) = \frac{\hat{U}\angle\phi}{2R_1}$, as the loop impedance is just $R_1 + R_2$ (equal to $2R_1$).
Converted to time-domain, with the same (cosine) reference, $i(t) = \frac{\hat{U}}{2R_1}\cos(\omega t + \phi)$.

d) The second coil is open-circuit, so it has no current; in the conventional mutual-inductors equation, $i_2 = 0$.
In this case, the first coil behaves exactly as it did before, since its mutual inductance term Mi_2 is zero.
So the current in the first coil is still i as was calculated in part 'c'.
The voltage across the second coil must therefore be $u_2 = j\omega Mi$.
The peak value of i is $\frac{\hat{U}}{2R_1}$, so the peak value of u_2 is $\frac{\omega M\hat{U}}{2R_1}$, which is is $\frac{\hat{U}\omega k\sqrt{LL_2}}{2R_1}$.

Q8

a) The impedances Z_1 are pure capacitors, so they consume no active power.
Each impedance Z_2 is $R + j\omega L$, and has a voltage $\sqrt{3}U$ applied to it.
The total complex power in the Δ-connected load is therefore $S_\Delta = \frac{3(\sqrt{3}U)^2}{Z_2^*} = \frac{9U^2}{R - j\omega L}$.
The real part of this is the active power, $P = \frac{9U^2 R}{R^2 + \omega^2 L^2}$, which is the total active power supplied by the source.

b) The reactive power into the Δ-connected load is $Q_\Delta = \frac{9U^2\omega L}{R^2 + \omega^2 L^2}$.
The reactive power into the Y-connected load is $Q_\mathsf{Y} = -3U^2\omega C$.
The two must sum to zero if no reactive power should come from the three-phase source, i.e. $\frac{9U^2\omega L}{R^2 + \omega^2 L^2} = 3U^2\omega C$.
This gives $C = \frac{3L}{R^2 + \omega^2 L^2}$.

c) When C is chosen to ensure no reactive power from the source, we know that each single-phase source is supplying a purely active power (no reactive power) equal to 1/3 of the total active power of the loads.
The current i_b will therefore have the same phase-angle as the voltage u_b.
Its magnitude will be $\frac{1}{3}P/U$, which is $\frac{1}{3U}\frac{9U^2 R}{R^2 + \omega^2 L^2}$.
So $i_\mathrm{b} = \frac{3UR}{R^2 + \omega^2 L^2}\angle -120°$.

Some questions during the exam

Q7. What are the meanings of aktiveffekt, genomsnittligeffekt and so on? *Aktiveffekt* is active power, which is an ac concept (real part of complex power). Mean power is the same thing, but suggestive of a more general time-domain concept. The question is expressed in the time-domain, but hints that you should use phasor analysis for an efficient solution.

Q7d. Is L_2 known? Yes; you can use L_2 as a known value. This should ideally have been pointed out. It was not included in the list at the start of Q7, as that was expected to cause extra confusion, when this component is not shown in the diagram. This detail has now been added to the text.

Q7. Carried-forward errors (följdfel) from part 'a' should not affect the grade in the later parts. However, the simplifications due to either solving 'a' correctly, or simply seeing the concept of L and C "cancelling" (series resonance when at maximum power condition), will make the later answers much simpler, which makes it easier for you to write them correctly.