Tentamen [s]: EI1102/EI1100 Elkretsanalys, 2014-10-31 kl 08–13

Denna tentamen är ett specialfall ('[s]') för ett fåtal studenter som har särskilt skäl att tentera kursen innan det nästa planerade tillfället, mars 2015. Den är baserad delvis på tentan för EI1110 HT14, och skrivit därför samtidigt.

Hjälpmedel: *Ett* A4-ark med studentens anteckningar (båda sidor). Dessutom, pennor.

Svar får anges på svenska eller engelska. En kort ordlista finns på sista sidan.
Tentan har 3 tal i del A (15p), och 2 tal i del B (15p).
Godkänt vid ≥25% på del A och del B individuellt, *och* ≥50% på delarna A och B tillsammans.
Betyget räknas från summan av A och B.

Läs varje tal noggrant **innan du försöker svara**.
Tänk på att **använda återstående tid till att kolla igenom varje svar**: man kan göra dimensionsanalys, rimlighetsbedömning (t.ex. "är det rätt att y går ner medan x går ner?"), och lösning genom en alternativ metod. Lösningar ska **förenklas** om inte annat är specificerat.
Satsa inte för mycket tid på bara en uppgift om du fastnar: ta hänsyn till poängvärden på uppgifterna, och att man måste både delar av tentan. Det är ofta så att **senare** *del*tal är betydligt **svårare** än de första deltalen.

Examinator: Nathaniel Taylor

Del A. Likström och Transienter.

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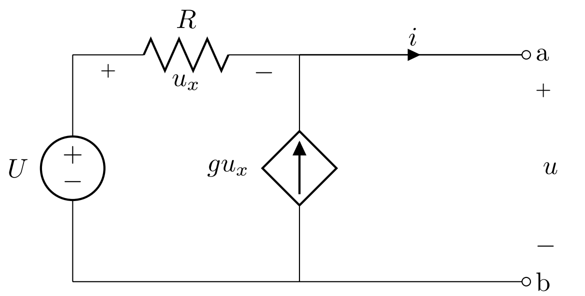
1) [5p]

a) [3p] Bestäm Theveninekvivalenten av kretsen här, med avseende till polerna 'a' och 'b'.

b) [2p] Vad är den största möjliga effekten som man kan få ut från denna krets, mellan polerna 'a' och 'b'? (Anta att g är positiv.)



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2) [5p]

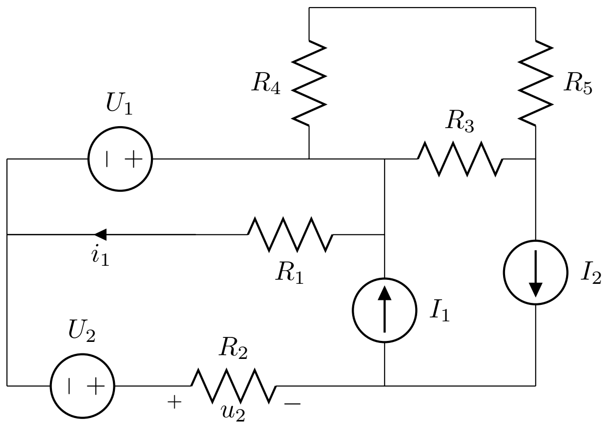
Bestäm de följande:

a) [1p] Strömmen i_1 (genom R_1).

b) [1p] Spänningen u_2 (över R_2).

c) [1p] Effekten leverad till R_3.

d) [2p] Effekten leverad från källan I_2.



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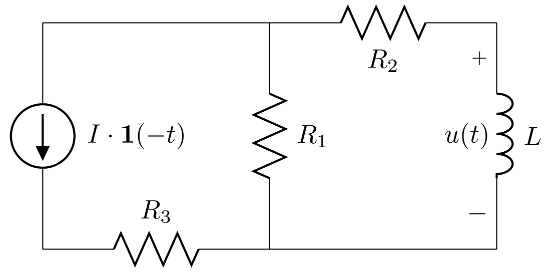
3) [5p]

Bestäm $u(t)$ för tider $t > 0$.

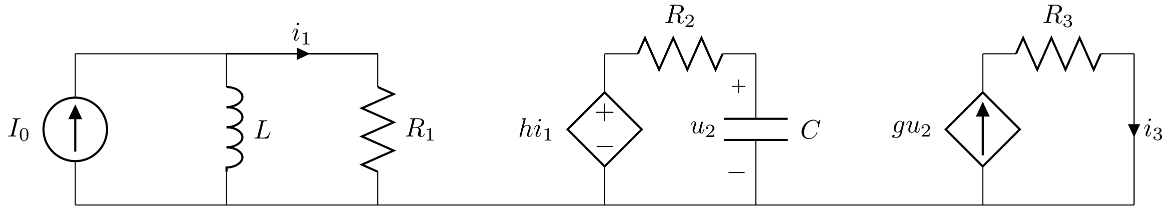
Enhetsstegfunktionen är $\mathbf{1}()$.
Observera minustecken i $\mathbf{1}(-t)$.
(Antag jämvikt innan steget, d.v.s. vid $t = 0^-$.)



Del B. Växelström

4) [8p]

a) [5p] Bestäm nätverksfunktion $\dfrac{i_3(\omega)}{I_0(\omega)}$ för den följande kretsen.



b) [1p] Visa att funktionen från deltal 'a)' kan skrivas i formen

$$H(\omega) = k \frac{\mathrm{j}\omega/\omega_1}{(1+\mathrm{j}\omega/\omega_1)(1+\mathrm{j}\omega/\omega_2)}.$$

c) [2p] Skissa ett Bodeamplituddiagram av funktionen $H(\omega)$ på antagandet $\omega_1 \ll \omega_2$, och $k = 1$. Markera viktiga punkter och lutningar.

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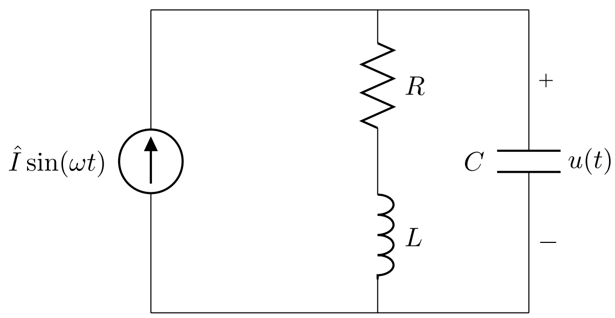
5) [7p]

a) [5p] Bestäm $u(t)$.

b) [1p] Bestäm C, som funktion av de andra storheterna R, L och ω, för att få maximaleffekt till motståndet R.

c) [1p] Vad *är* maximaleffekten i situationen av deltal 'b)'? (Förenkla svaret.)



Solutions (EI1102/EI1100, Tenta[special] 2014-10-31)

Q1, Q2, Q3

For questions 1, 2, 3, see (`http://www.etk.ee.kth.se/ei1110/exams/`) the EI1110 exam that was simultaneous with this one.

The questions there have different numbering, but there are two that are identical to Q2 and Q3 here, and one that has a part identical to Q1a here.

There is a difference in Q1b. The maximum *power* is requested in this exam, but the EI1110 exam requested the *current* corresponding to the maximum-power condition. The maximum power itself can be found by considering that the maximum power point is when the current is half of the short-circuit current, and the voltage half of the open-circuit voltage. The power from the circuit is then $\frac{1}{4}u_{\mathrm{oc}}i_{\mathrm{sc}}$ which is $\frac{1}{4}\frac{u^2_{\mathrm{Thevenin}}}{R_{\mathrm{Thevenin}}}$, $P_{\max} = \frac{U^2(1+gR)}{4R}$.

Q4

a) Three separate parts of the circuit are joined to each other only by one node. There therefore cannot be currents between these parts: they can be analysed separately. (Only the potentials are forced to have a relation due to the single node connecting the parts. In this analysis we don't need to use potentials, and haven't even chosen a ground node. Only the voltages and currents need be considered.)

By current division in the left part,

$$i_1 = I_0 \frac{\mathrm{j}\omega L}{R_1 + \mathrm{j}\omega L}.$$

By voltage division in the middle part,

$$u_2 = h i_1 \frac{\frac{1}{\mathrm{j}\omega C}}{R_2 + \frac{1}{\mathrm{j}\omega C}}.$$

In the right part, the resistor is irrelevant to the solution: the marked current is in series with a current source, so it is determined directly.

$$i_3 = g u_2.$$

Putting these together gives the full network function,

$$\frac{i_3}{I_0} = \frac{i_1}{I_0}\cdot\frac{u_2}{i_1}\cdot\frac{i_3}{u_2} = gh\cdot\frac{\mathrm{j}\omega L}{R_1+\mathrm{j}\omega L}\cdot\frac{\frac{1}{\mathrm{j}\omega C}}{R_2+\frac{1}{\mathrm{j}\omega C}} = gh\cdot\frac{\mathrm{j}\omega L/R_1}{1+\mathrm{j}\omega L/R_1}\cdot\frac{1}{1+\mathrm{j}\omega C R_2}.$$

b) Taking the final part of the solution from part 'b)', we can write
$k = gh$, $\omega_1 = \frac{R_1}{L}$, $\omega_2 = \frac{1}{CR_2}$.
Comparing with 'a)', we see that the required property is acheieved: $H(\omega) = k\frac{\mathrm{j}\omega/\omega_1}{(1+\mathrm{j}\omega/\omega_1)(1+\mathrm{j}\omega/\omega_2)} = \frac{i_3(\omega)}{I_0(\omega)}$.

c) Draw a band-pass filter: flat (0 dB/decade) when $\omega_1 < \omega < \omega_2$, and falling away at 20 dB/decade when the frequency goes above or below this range. You should mark the level of the flat part as $20\,\log_{10}(k)$ dB on the plot.

(Explanation: The the two terms with ω_1 cancel each other when $\omega \gg \omega_1$. The term with ω_2 is just $\simeq 1$ when $\omega \ll \omega_2$. so the result is 1, which means 0 dB. Therefore, the flat part would have a gain of 0 dB if k is ignored. Multiplying this by k causes the dB value of k to be added to the dB gain of $H(\omega)$.)

Q5

a) The question should pedantically have said something about "in steady state" or "for the forced response after transients have died away". It seems we don't check quite so carefully when the exam is only taken by two or three people! Anyway, the above assumptions are conventional from past exams, when a time-waveform is to be found in a circuit with a sinusoidal source.

Let us use ac analysis: "jω-metoden". Let us take a sine reference with peak values, and therefore represent the source as $I(\omega) = \hat{I}\underline{/0}$.

The voltage marked $u(t)$ is across the capacitor, which is parallel with the source and with the branch of the resistor and inductor. This is therefore the same voltage as across the source. Its frequency-domain value,

$u(\omega)$, can be found by multiplying the source current by the total impedance of the three components that are connected to the source,

$$u(\omega) = I(\omega)\,\frac{(R+\mathrm{j}\omega L)\,\frac{1}{\mathrm{j}\omega C}}{(R+\mathrm{j}\omega L)+\frac{1}{\mathrm{j}\omega C}} = I(\omega)\,\frac{R+\mathrm{j}\omega L}{1-\omega^2 LC+\mathrm{j}\omega CR}.$$

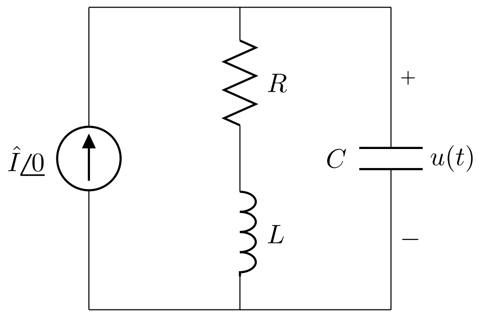
To write the time-function $u(t)$, we must find its amplitude and phase, which are the magnitude and angle of this function $u(\omega)$. If we want to keep the working simple, we can handle the magnitudes and angles of the numerator separately.
Note: an error in the term $(1-\omega^2 LC)^2$ in the expression below has been corrected 2015-05-28: thanks!

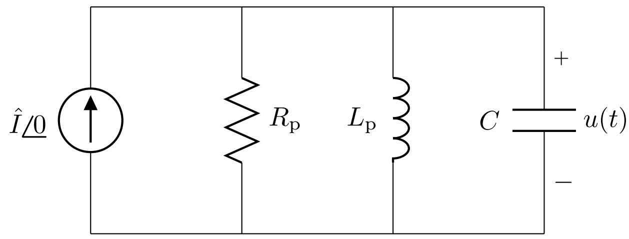
$$u(t) = \frac{\hat{I}\sqrt{R^2+\omega^2 L^2}}{\sqrt{\left(1-\omega^2 LC\right)^2+\omega^2 C^2 R^2}}\ \underline{\big/ 0+\tan^{-1}\frac{\omega L}{R}-\tan^{-1}\frac{\omega CR}{1-\omega^2 LC}}.$$

It could be argued to look neater if the $\tan^{-1}$ terms were combined, e.g. by first making the denominator of $u(\omega)$ purely real. But then the argument to the $\tan^{-1}$ function would be nastier. We'll leave it as it is!

b) This is a bit of an unusual "maximum power" question, because the 'load' is fixed.

Perhaps the clearest approach is to convert the series branch L and R to a parallel pair L_p and R_p that is equivalent (for sinusoidal steady state at angular frequency ω).





If these alternative pairs of components are equivalent, then their admittances Y must be equal,

$$Y = \frac{1}{R_\mathrm{p}}+\frac{1}{\mathrm{j}\omega L_\mathrm{p}} = \frac{1}{R+\mathrm{j}\omega L} = \frac{R-\mathrm{j}\omega L}{R^2+\omega^2 L^2},$$

from which, by equating real and imaginary terms, $R_\mathrm{p} = \frac{R^2+\omega^2 L^2}{R}$ and $L_\mathrm{p} = \frac{R^2+\omega^2 L^2}{\omega^2 L}$. The resistor R_p in the circuit on the right is still the only way that active power can be consumed in the circuit. Maximising power into R_p should therefore maximise power into R in the original circuit.

When the reactances of L_p and C are made to be equal in magnitude (they will be opposite in sign, as one is capacitive and one is inductive), these two components in parallel appear as an open circuit: this condition maximises the current through R_p and therefore its power. This requires $\frac{1}{\omega C} = \omega L_\mathrm{p} = \frac{R^2+\omega^2 L^2}{\omega L}$ from which

$$C = \frac{L}{R^2+\omega^2 L^2}.$$

c) It's most convenient to calculate this by using the parallel equivalent of the series R-L branch. Then we know that in the maximum power condition the inductor will resonate with the parallel capacitor, so all current $I(\omega)$ goes through the parallel resistance, $\frac{R^2+\omega^2 L^2}{R}$. The power, bearing in mind that $\hat{I}$ is the peak value of $I(\omega)$, is then

$$P_\mathrm{max} = \frac{\hat{I}^2}{2}\,\frac{R^2+\omega^2 L^2}{R}$$