EI1120 Elkretsanalys, Kontrollskrivning KS1, 2015-02-04 kl 08–10

Hjälpmedel: Ett A4-ark med godtyckligt innehåll (handskriven, datorutskrift, diagram, m.m.).

Kontrollskrivningen har 3 tal, med totalt 12 poäng. Den omfattar ämnet 'Likström' och motsvarar del A i tentamen. Det högre av betygen från KS1 (den här) och från tentans del A kommer att användas vid betygsättning av tentan. Del A är godkänt vid ≥40%, men glöm inte att tentan kräver också minst 50% räknat över talen i alla delar.

Om inte annan information anges i ett tal, ska: angivna värden av komponenter (t.ex. R för en motstånd, U för en spänningskälla) antas vara kända storheter; och andra storheter (t.ex. strömmen markerad i en motstånd) antas vara okända storheter; och komponenter antas vara idéala.
Lösningar ska uttryckas i kända kvantiteter, och förenklas. Var tydlig med diagram och definitioner av variabler. Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Använd återstående tid för att kolla på svaren! Examinator: Nathaniel Taylor

---

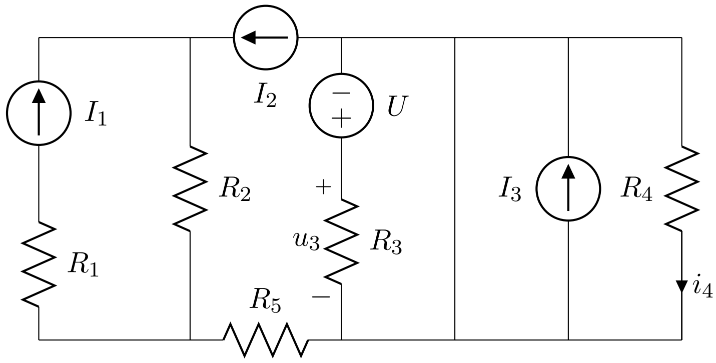
1) [4p]

a) [1p] Bestäm strömmen i_4.

b) [1p] Bestäm spänningen u_3.

c) [1p] Vilken effekt tar R_2?

d) [1p] Vilken effekt levereras av källan U?

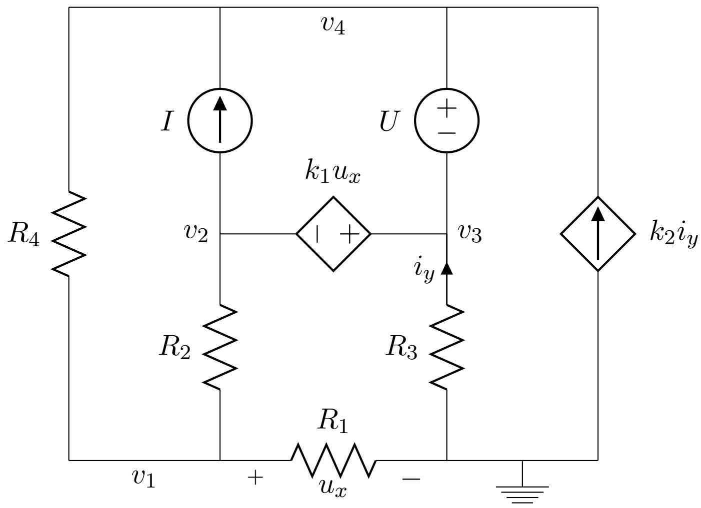


---

2) [4p]

Använd nodanalys för att skriva ekvationer som skulle kunna lösas för att få ut de markerade nodpotentialerna v_1, v_2, v_3, v_4.

Du behöver bara visa att du kan översätta från kretsen till ekvationerna: du *måste inte* lösa eller förenkla ekvationerna.

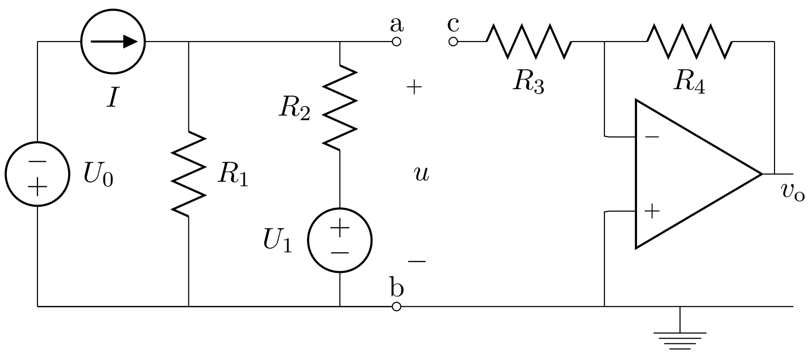


---

3) [4p]

a) [3p] Bestäm Theveninekvivalenten sett mellan polerna 'a' och 'b', d.v.s. där spänningen u är markerad. (Komponenterna R_3, R_4 och opampen försummas, då 'a'-'c' är öppet.)

b) [1p] Polerna 'a' och 'c' kopplas nu ihop (kortslutna). Vad blir potentialen v_o då?



Solutions, EI1120 KS1 2015-02-04

1) Circuit quick-puzzles.

a) $i_4 = 0$, by Ohm's law, as R_4 is parallel with a short-circuit (0 V)

b) $u_3 = U$, by KCL around the loop of R_3, U and short-circuit: $u_3 + (-U) + 0 = 0$.

c) $P_{R_2} = i_{R_2}^2 R_2 = (I_1 + I_2)^2 R_2$, by KCL above R_2.

d) $P_U = U i_U = \frac{U^2}{R_3}$, see answer to part 'b)'; one has to be careful about the sign of the current.

2)

Extended nodal analysis ("the simple way")

Let's define the unknown currents in the voltage sources, with the positive direction going into the source's + terminal: call it i_α in source U, and i_β in source $k_1 u_x$.

KCL (outgoing currents) at all nodes except ground:

$$\text{KCL}(1): \quad 0 = \frac{v_1 - v_4}{R_4} + \frac{v_1 - v_2}{R_2} + \frac{v_1}{R_1} \tag{1}$$

$$\text{KCL}(2): \quad 0 = \frac{v_2 - v_1}{R_2} + I - i_\beta \tag{2}$$

$$\text{KCL}(3): \quad 0 = \frac{v_3}{R_3} + i_\beta - i_\alpha \tag{3}$$

$$\text{KCL}(4): \quad 0 = \frac{v_4 - v_1}{R_4} - I + i_\alpha - k_2 i_y \tag{4}$$

Now there are 8 unknowns $(v_1, v_2, v_3, v_4, i_\alpha, i_\beta, u_x, i_y)$, but only 4 equations.

Next, include the further information given by the voltage sources,

$$v_4 - v_3 = U \tag{5}$$

$$v_3 - v_2 = k_1 u_x \tag{6}$$

and also define the controlling variables u_x and i_y in terms of the existing known or unknown quantities,

$$u_x = v_1 - 0 \tag{7}$$

$$i_y = -\frac{v_3}{R_3} \tag{8}$$

There are now 8 equations in 8 unknowns. The systematic way in which this was done is important! There *are* plenty of ways to write a sufficient set of equations, but it is also dangerously easy to write some linearly dependent equations and assume that "n unknowns, n equations, therefore it's all ok" ... the above method is useful!

Computer-assisted check of solutions to Question 2

We can choose some arbitrary numeric values:

$U = 12\,\text{V}, \quad I = 1\,\text{A}, \quad k_1 = 0.4, \quad k_2 = 3, \quad R_1 = 6\,\Omega, \quad R_2 = 18\,\Omega, \quad R_3 = 30\,\Omega, \quad R_4 = 5\,\Omega$

and then compare solutions of our equations and a circuit-solver program.

The circuit can be described by the following "netlist" for solving in SPICE. (The extra node 5, and zeroed voltage source `Vix`, are the way of measuring the current i_y to control CCCS `F1`.)

```
EI1120_VT15_KS1Q2
V1    4 3  DC  12.0
I1    2 4  DC   1.0
R1    1 0       6.0
R2    2 1      18.0
R3    3 5      30.0
Viy   0 5  DC   0.0
R4    4 1       5.0
E1    3 2  1 0  0.4
F1    0 4  Viy  3.0
.OP
.PRINT DC V(0) V(1) V(2) V(3) V(4)
.END
```

The result from putting the above into SPICE 2g.6 [1983-03-15!] is

```
 node potentials:
  (  1)    3.1418
  (  2)   -5.1840
  (  3)   -3.9273
  (  4)    8.0727
  (  5)    0.0000
 voltage source currents:
    v1        4.065E-01
    viy       1.309E-01
 total power dissipation:
    8.38E+00  watts
 voltage-controlled voltage sources:
   0           e1
    v-source   1.257
    i-source   5.37E-01
 current-controlled current sources:
   0           f1
    i-source   3.93E-01
```

Putting our equation systems directly into Matlab symbolic toolbox, the 'extended method' gives

```
%% solve the equation-system symbolically, for the 6 listed unknowns
% (put an underscore onto variable "I", to avoid it being treated as an imaginary unit
% in matlab symbolic toolbox)
s = solve( ...
  '0 =  (v1-v4)/R4 + (v1-v2)/R2 + v1/R1', ...
  '0 =  (v2-v1)/R2 + I_ - ibeta', ...
  '0 =  v3/R3 + ibeta - ialpha', ...
  '0 =  (v4-v1)/R4 - I_ + ialpha - k2*iy', ...
  'v4 - v3 = U', ...
  'v3 - v2 = k1*ux', ...
  'ux = v1 - 0', ...
  'iy = -v3/R3', ...
  'v1, v2, v3, v4, ialpha, ibeta, ux, iy' )

%%  if you want, show all the long expressions for the 8 unknowns:
for f=fields(s)', disp(f{1}), simplify(s.(f{1})), end

%% set numeric values for comparison with the SPICE solution
 U = 12,  I_ = 1,  k1 = 0.4,  k2 = 3,  R1 = 6,  R2 = 18,  R3 = 30,  R4 = 5
% find the result of substituting the above values into the symbolic expressions
for f=fields(s)', fprintf(' %s: %f\n', f{1}, double(subs(s.(f{1}))) ); end
```

```
     v1:  3.141818
     v2: -5.184000
     v3: -3.927273
     v4:  8.072727
 ialpha:  0.406545
  ibeta:  0.537455
     iy:  0.130909
     ux:  3.141818
```

Fortunately, this matches the earlier solution (after correcting a mistake in the SPICE input file!).

---

3)

a) The source U_0 is irrelevant, as it's in series with a current source and we don't want to find any quantities *within* this branch.

Nodal analysis (KCL at a single node) gives, for the open-circuit situation,

$$-I + \frac{u}{R_1} + \frac{u - U_1}{R_2} = 0,$$

whence

$$u_{\text{oc}} = U_{\text{T}} = \frac{(U_1 + IR_2)\,R_1}{R_1 + R_2}.$$

The Thevenin resistance (source resistance) can be found by short-circuit current, or by setting the sources I and U_1 (and U_0 if we haven't noticed that it has no influence) to zero and calculating the equivalent resistance of the circuit. Using the latter approach, we have R_1 and R_2 in parallel, so

$$R_{\text{T}} = \frac{R_1 R_2}{R_1 + R_2}.$$

You really *should* then draw the diagram showing the Thevenin voltage and resistance with the right direction of voltage with respect to the marked terminals!

b) The usual inverting-amplifier formula can easily be derived for the relation $\frac{v_{\text{o}}}{v_{\text{c}}} = \frac{-R_4}{R_3}$. But we can't just use this together with the open-circuit potential $v_a = U_{\text{T}}$, to find v_{o}. In this circuit, $v_{\text{o}} \neq \frac{-R_4}{R_3} U_{\text{T}}$. That's because this opamp circuit does not have an infinite input resistance at terminal 'c', and the earlier circuit (left) does not have a zero output resistance at point 'a'.

If a potential v is put on terminal 'c', then a current $\frac{v}{R_3}$ will flow from 'c' to the virtual earth node of the opamp's inverting input, as the non-inverting input is connected to the earth node (current into the virtual earth node leaves it through R_4 ... the opamp's input terminal has no current). And, if a current is drawn from terminal 'a', the voltage u will change compared to its open-circuit value. So when the terminals 'a'-'c' are connected (shorted together) a current will flow between them, and the voltage u will depend on the Thevenin voltage and resistance of the circuit on the left *and* on the input resistance of the opamp circuit on the right.

With 'a'-'c' connected, we find (by using the Thevenin equivalent and voltage division between R_{T} and R_3) that

$$u_{\text{ab}} = a_{\text{cb}} = v_{\text{c}} = U_{\text{T}} \frac{R_3}{R_{\text{T}} + R_3}.$$

The inverting amplifier gain $\frac{-R_4}{R_3}$ *can* be used if we've correctly worked out what v_{c} is for the complete circuit:

$$v_{\text{o}} = U_{\text{T}} \cdot \frac{-R_4}{R_3} \cdot \frac{R_3}{R_{\text{T}} + R_3} = -\frac{(U_1 + IR_2)\,R_1}{R_1 + R_2} \cdot \frac{R_4}{R_3} \cdot \frac{R_3}{\frac{R_1 R_2}{R_1 + R_2} + R_3} = -\frac{(U_1 + IR_2)\,R_1 R_4}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$