EI1110 Elkretsanalys (Elektro) Tentamen TEN1x, 2015-10-26 kl 14–19

Anmärkning: TEN1 för de flesta Elektro studenter är bokad på 2015-10-29. TEN1x (2015-10-26) är ett extra tillfälle, betraktat som likvärdigt, för ett fåtal studenter som har en annan tenta på den 29:e.

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt.

Tentan har 5 tal i två sektioner: 3 i sektion A (12p), och 2 i sektion B (10p).

Godkänd tentamen TEN1 kräver:

$$\frac{\max(a, a_\mathrm{k})}{A} \geq 40\% \qquad \& \qquad \frac{b}{B} \geq 40\% \qquad \& \qquad \frac{\max(a, a_\mathrm{k}) + b}{A + B} \geq 50\%$$

där A=12 och B=10 är de maximala möjliga poängen från sektionerna A och B, a och b är poängen man fick i dessa respektive sektioner i tentan, och a_k är poängen man fick från kontrollskrivning KS1 vilken motsvarar tentans sektion A; funktionen max() tar den högre av sina argument.

Betyget räknas från summan över båda sektioner, igen med bästa av sektion A och KS1, $\frac{\max(a,a_\mathrm{k})+b}{A+B}$. Betygsgränserna (%) är 50 (E), 60 (D), 70 (C), 80 (B), 90 (A).

I vissa gränsfall där betyget är lite under 50%, eller bara en av sektionerna är underkänd trots 50% eller bättre totalt, kommer betyget 'Fx' registreras, med möjlighet att få betyget E om ett kompletteringsarbete är godkänt inom några veckor efter tentamen.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara **kända** storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller en spänningskälla) antas vara **okända** storheter.

Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner.

Dela tiden mellan talen — senare deltal brukar vara svårare att tjäna poäng på ... fastna inte!

Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Använd kvarstående tid för att kontrollera svaren. Lycka till! Nathaniel Taylor (073 919 5883)

Sektion A. Likström

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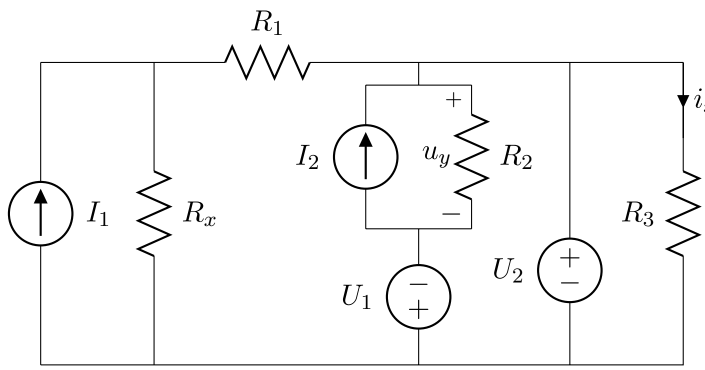
1) [4p]

a) [3p] Bestäm de följande:
De markerade i_x och u_y.
Effekten som källan I_2 levererar.
Effekten som motståndet R_1 får.

b) [1p] Vilket värde ska R_x ha för att få den största möjliga effekten från resten av kretsen? (Uttryck som funktion av andra komponentvärden.)

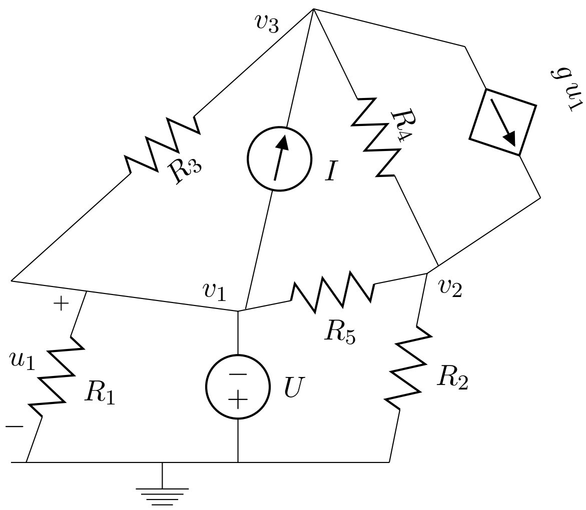


2) [4p]

Skriv ekvationer som skulle kunna lösas för att finna de markerade potentialerna v_1, v_2 och v_3, som funktioner av de givna komponentvärdena.

Du **måste inte** lösa eller förenkla dina ekvationer.

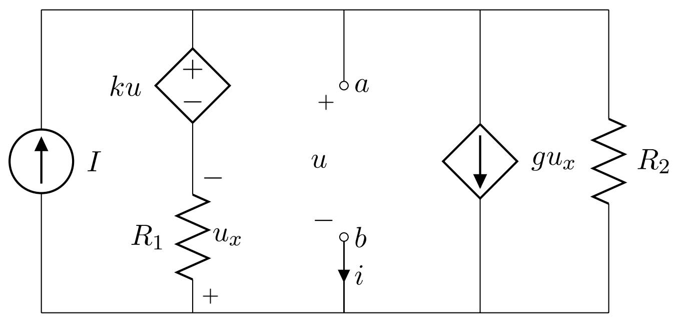
Använd helst en systematisk metod, för att försäkra tillräckliga ekvationer utan onödigt arbete.



3) [4p]

Bestäm Nortonekvivalanten med avseende på polerna a och b.

Ledning: Ett bra sätt är att bestämma sambandet mellan u och i för kretsen här samt för en Nortonekvivalent, och därigenom att identifiera Nortonparametrarna. Metoden med kortslutning och öppenkrets fungerar också.



Sektion B. Transient

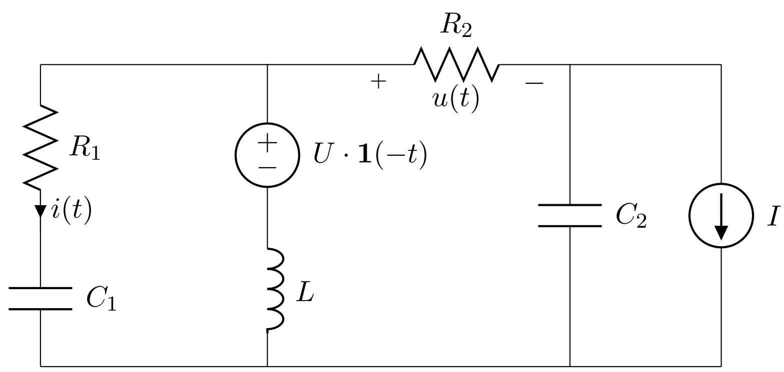
4) [5p]

Bestäm $u(t)$ och $i(t)$ vid de följande tiderna:

a) [2p] $t = 0^-$

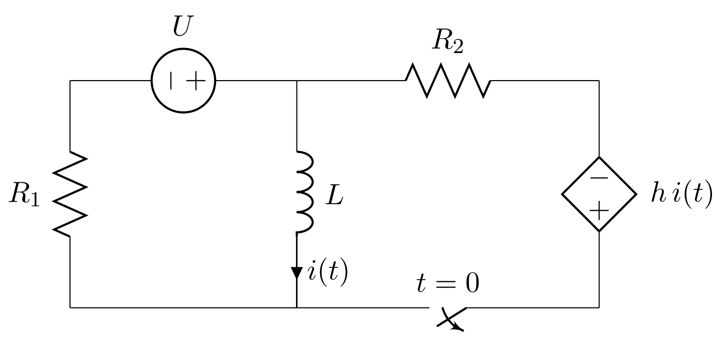
b) [2p] $t = 0^+$

c) [1p] $t \to \infty$



5) [5p]

Bestäm $i(t)$ för tider $t > 0$.



Solutions, EI1110 TEN1x 2015-10-26

1)

a)

$i_x = U_2 \,/\, R_3$.

$u_y = U_1 + U_2$.

$P_{\text{I2}} = (U_1 + U_2)\ I_2$.

$P_{\text{R1}} = \left(\frac{U_2 - I_1 R_x}{R_1 + R_x}\right)^2 R_1$.

b) This is clearly a maximum power question. Now R_x is the component that is to be chosen, and the rest of the circuit is fixed. The most obvious way is to find the equivalent resistance (output resistance, Thevenin resistance, Norton resistance … lots of names) of the rest of the circuit, seen from the nodes where R_x is connected (without including R_x). Then, by the maximum power theorem, R_x needs to be chosen to equal this value. There is no dependent source, so the equivalant resistance of the circuit is easiest to find by setting the sources to zero.

Steps: remove R_x; set all sources to zero; find the resistance between the points where R_x was connected. The source U_2 shorts the other branches of R_3 and U_1 etc. The source I_1 is open-circuit. So the equivalent resistance is just R_1.

$R_x = R_1$ for maximum power transfer.

2) Two solution methods are shown, and a numerical check is made.

Extended nodal analysis ("the simple way")

Let's define the unknown current in voltage source U as i_α, going into the source's + terminal.

KCL (outgoing currents) at all nodes except ground:

$$\text{KCL}(1): \quad 0 = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_5} + \frac{v_1 - v_3}{R_3} + I - i_\alpha \tag{1}$$

$$\text{KCL}(2): \quad 0 = \frac{v_2}{R_2} + \frac{v_2 - v_1}{R_5} + \frac{v_2 - v_3}{R_4} - g u_1 \tag{2}$$

$$\text{KCL}(3): \quad 0 = \frac{v_3 - v_1}{R_3} + \frac{v_3 - v_2}{R_4} - I + g u_1 \tag{3}$$

Next, include the further information given by the voltage source,

$$v_1 = -U, \tag{4}$$

and then describe how the marked u_1 (the controlling variable for the VCCS) is defined,

$$u_1 = v_1 - 0 = v_1. \tag{5}$$

Alternative: Supernode method

KCL is done at each node (or supernode group) apart from the ground node or supernode. We have just one voltage source, U. It is between ground and the node marked v_1. Thus, the node of v_1 becomes part of the ground supernode, and KCL is not applied to it.

The following equation describes what the voltage source does in the circuit. It is needed in our answer, in order to define v_1.

$$v_1 = -U \tag{1}$$

Now write KCL at all other nodes or supernodes except the ground one: this is in fact just two in our case! We use the above equation to avoid using the symbol v_1, so that there will only be two unknowns. We also express the controlling variable as $u_1 = v_1 = U$ in order to avoid the unknown u_1.

$$\text{KCL}(2): \quad 0 = \frac{v_2}{R_2} + \frac{v_2 + U}{R_5} + \frac{v_2 - v_3}{R_4} + gU \tag{2}$$

$$\text{KCL}(3): \quad 0 = \frac{v_3 + U}{R_3} + \frac{v_3 - v_2}{R_4} - I - gU \tag{3}$$

3)

This is a good situation for KCL, as we have 5 parallel branches between two nodes:

$$-I + \frac{u - ku}{R_1} + i + gu_x + \frac{u}{R_2} = 0.$$

The controlling variable u_x can be expressed as $u_x = ku - u = (k-1)u$. Substituting this, and rearranging the KCL equation,

$$i = I - \left(\frac{1}{R_2} + (k-1)\left(g - \frac{1}{R_1}\right)\right) u.$$

Comparing this to the i-u relation of a Norton source,

$$i = I_{\text{N}} - \frac{u}{R_{\text{N}}},$$

we identify $I_{\text{N}} = I$ and $R_{\text{N}} = \frac{1}{\frac{1}{R_2} + (k-1)(g - 1/R_1)}$, which could be 'simplified' to $R_{\text{N}} = \frac{R_1 R_2}{R_1 + (k-1)(gR_1R_2 - R_2)}$.

The Norton source *should be drawn* to show the right components, right connections, and right direction relative to the marked terminals.

4)

The only change in the circuit is at $t = 0$: the voltage source changes from U to 0 at this time.

a) Initial equilibrium, $t = 0^-$.

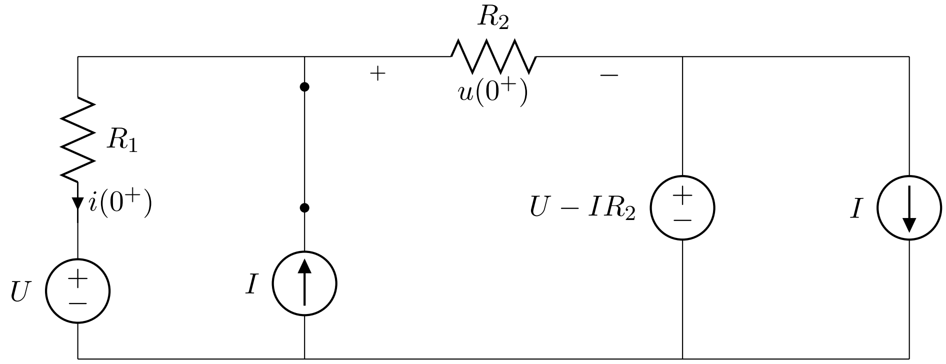
Re-draw the diagram. The voltage source has value U. The capacitors are replaced with open-circuits, as no current flows in them if the voltage is constant (steady). The inductors are replaced with short-circuits, as no voltage appears across them if the current is constant.

Notice then that $i(0^-) = 0$ since the open-circuit C_1 is in series.

With the two open-circuits, the remaining circuit is a single loop, so the current I has to flow through R_2 in a direction that gives $u(0^-) = IR_2$.

b) Immediately after the step-change: continuity.

The continuous variables of the three reactive components can be found from $t = 0^-$. Using the principle of continuity, the reactive components can then be replaced by sources with these values, for $t = 0^+$. This is shown below, with the voltage source also replaced with its new value of zero (short-circuit).



Notice that the actual current-source I (the rightmost branch) is irrelevant to the two variables we're looking for: there is a voltage-source in parallel with it ($U - IR_2$, the voltage on C_2).

It is now possible to do KCL on the node about R_1: let us call the potential v, relative to the bottom node. The solution for v ends up being a short one!

$$\frac{v-U}{R_1} - I + \frac{v-U+IR_2}{R_2} = 0 \qquad \Longrightarrow \qquad v = U.$$

From this, we can then find u and i at this time: $i = \frac{v-U}{R_1} = 0$ and $u = v - U + IR_2 = IR_2$.

Hence $i(0^+) = 0$, and $u(0^+) = IR_2$.

c) Final equilibrium, $t \to \infty$.

The only difference from the circuit at $t = 0^-$ is that the voltage source is zero, i.e. can be seen as a short-circuit.

Still, $i(\infty) = 0$, as the capacitor carries no current in equilibrium (modelled as open-circuit).

Still, $u(\infty) = IR_2$, as the circuit is a single loop with current I passing around it. (The voltage source is now zero, but this doesn't affect the voltage u: the current source sets the current in the loop, and its voltage changes at $t = 0$ to match the change that happened in the voltage source.)

(**A note.** Perhaps that sounds strange: three time-points, all with the same solution, in spite of a source-value having a step-change. There *are* changes in the circuit quantities: the voltages on both capacitors do change between $t = 0$ and $t \to \infty$. But the only thing that happens initially (at $t = 0^+$) is that the change of voltage-source value is met with a change of the inductor's voltage: seen from outside this branch of voltage source and inductor, the branch has not immediately started behaving differently. However, this voltage now across the inductor causes its current to start changing. This in turn leads to changes in the voltages on the capacitors, untill the new equilibrium is approached. Depending on the actual component values, this transition may be achieved by a slow smooth change, or a dying oscillation.)

5)

When $t > 0$ the switch is open: then only the loop at the left is relevant to $i(t)$. That loop is like a Thevenin source connected to an inductor. Its time-constant is therefore $\tau = L/R_1$. The final value of i is $i(\infty) = U/R_1$.

What's more effort is the initial condition, $i(0^+)$. Continuity can be used to show this is equal to $i(0^-)$, which can be found by equilibrium in the circuit before the switch opens. For this equilibrium, the inductor can be represented as a short-circuit. KCL gives

$$i - \frac{U}{R_1} + \frac{hi}{R_2} = 0, \qquad \Longrightarrow \qquad i(0^-) = \frac{R_2}{h+R_2} \cdot \frac{U}{R_1}.$$

Putting together the initial value, final value and time-constant, we find

$$i(t) \;=\; \frac{U}{R_1} + \left(\frac{R_2}{h+R_2} \cdot \frac{U}{R_1} - \frac{U}{R_1}\right) \mathrm{e}^{-tR_1/L} \;=\; \frac{U}{R_1}\left(1 - \frac{h}{h+R_2}\,\mathrm{e}^{-tR_1/L}\right) \qquad (t > 0).$$