EI1120 Elkretsanalys, Kontrollskrivning KS1, 2016-02-05 kl 13–15

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt.

Kontrollskrivningen har 3 tal, med totalt 12 poäng. Den omfattar ämnet 'Likström' och motsvarar sektion A i tentan. I tentan är kravet för godkänd minst 40% för sektion A, samt minst 50% över hela tentan.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter.
Lösningar ska uttryckas i kända storheter, och förenklas.
Var tydlig med diagram och definitioner av variabler.
Dela tiden mellan talen — senare deltal brukar vara svårare att tjäna poäng på ... fastna inte på dessa.
Kontrollera svarens rimlighet genom t.ex. dimensionskontroll eller alternativ lösningsmetod.

Använd återstående tid för att kontrollera svaren!

Nathaniel Taylor (073 919 5883)

---

1) [4p]

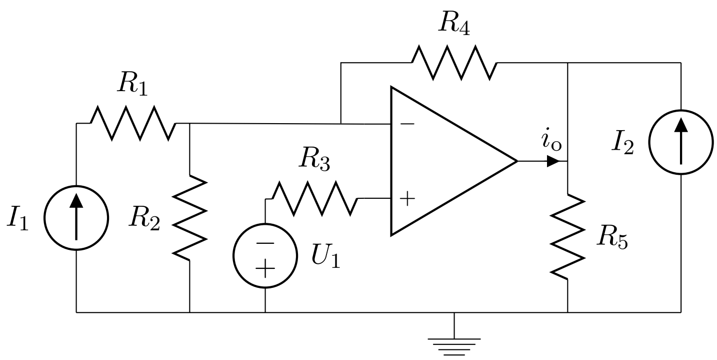
Bestäm följande:

a) [1p] Effekten levererad av källen U_1.

b) [1p] Effekten absorberad av R_1.

c) [1p] Effekten absorberad av R_5.

d) [1p] Strömmen markerad i_o.

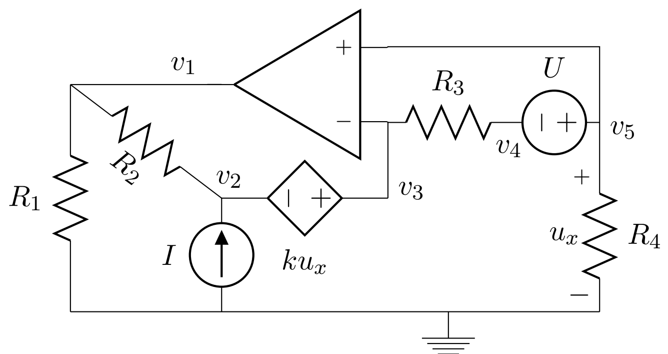


---

2) [4p]

Använd nodanalys för att skriva ekvationer som *skulle kunna lösas* för att få ut de markerade nodpotentialerna v_1, v_2, v_3, v_4 och v_5.

Du *måste inte* lösa eller förenkla ekvationerna: du behöver bara visa att du kan översätta från kretsen till ekvationerna.

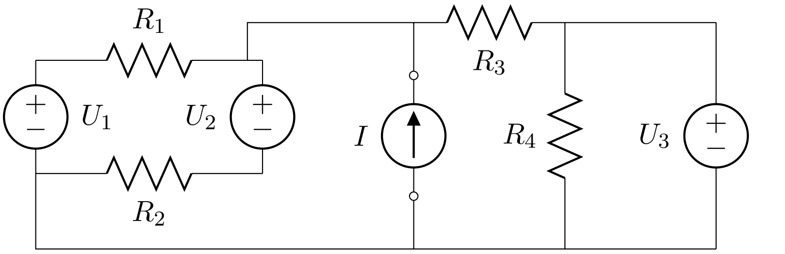


---

3) [4p]

Det finns ett värde av strömmen I som maximerar effekten som levereras till strömkällen från resten av kretsen.

Antag att:
$U_1 = U_2 = U_3 = U$, och
$R_1 = R_2 = R_3 = R_4 = R$.
Bara U och R ska användas i svaren.



a) [3p] Uttryck detta värde av I som funktion av U och R.

b) [1p] Vilken effekt levereras då till strömkällen?

Solutions, EI1120 KS1 2016-02-05

1)

a) Power supplied by the source U_1: $\boxed{0}$ (opamp input has zero current)

b) Power absorbed by R_1: $\boxed{I_1^2 R_1}$ (R_1 is in series with current source I_1)

c) Power absorbed by R_5: $\boxed{\dfrac{\left(I_1 R_4 + U_1\left(1+\frac{R_4}{R_2}\right)\right)^2}{R_5}}$

Method: solve for the opamp's output potential (let's call it v_o); then the requested power is $\frac{v_o^2}{R_5}$.
So, how can we find v_o?
The potential at the inverting input is $-U_1$: this is found from seeing that with no current through R_3, the potential at the *non-inverting* input must be $-U_1$; we then assume the *inverting* input is held to this value by the negative feedback.
KCL at the inverting input then gives an equation where v_o is the only unknown. A possible simplification is to notice that R_1 is irrelevant (in series with a current source), so I_1 and R_2 can be source-transformed into a Thevenin source with voltage $I_1 R_2$ and resistance R_2. But straight nodal analysis is probably just as good or better:

$$\frac{-U_1 - v_o}{R_4} + \frac{-U_1}{R_2} - I = 0 \quad \Longrightarrow \quad v_o = -I_1 R_4 - U_1\left(1 + \frac{R_4}{R_2}\right).$$

d) The opamp output current: $\boxed{i_o = -\frac{R_2+R_4+R_5}{R_2 R_5} U_1 - \left(1+\frac{R_4}{R_5}\right) I_1 - I_2}$

Method: using v_o from before, the currents in R_4 and R_5 can be found, then KCL can be applied to the node of the opamp output to obtain

$$i_o = \frac{v_o}{R_5} + \frac{v_o - (-U_1)}{R_4} - I_2.$$

Substituting the known expression for v_o (see part 'c') then rearranging into coefficients of U_1, I_1 and I_2, the final expression for i_o is found. It's not obvious what is the simplest way to express this, so many variations (rearrangements) of the solution would be acceptable.

---

2) Two examples will be shown. Many variations are possible. The first example is the one that we suggest is probably easiest to do for this type of question, based on systematic use of simple rules.

Extended nodal analysis ("the simple way")

We'll define the unknown currents in the two voltage-sources to be going into the source's + terminal. We'll call them i_α in the independent source U, and i_β in the dependent source hi_y. The current out of the opamp's output can be called i_o.

First we write KCL at all nodes except ground:

$$\text{KCL}(1)_{(\text{out})}: \quad 0 = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} - i_o \tag{1}$$

$$\text{KCL}(2)_{(\text{out})}: \quad 0 = \frac{v_2 - v_1}{R_2} - i_\beta - I \tag{2}$$

$$\text{KCL}(3)_{(\text{out})}: \quad 0 = i_\beta + \frac{v_3 - v_4}{R_3} \tag{3}$$

$$\text{KCL}(4)_{(\text{out})}: \quad 0 = \frac{v_4 - v_3}{R_3} - i_\alpha \tag{4}$$

$$\text{KCL}(5)_{(\text{out})}: \quad 0 = \frac{v_5}{R_4} + i_\alpha. \tag{5}$$

These are only 5 equations so far, but with 8 unknowns: v_1, v_2, v_3, v_4, v_5, i_α, i_β, i_o.

We can add the further information given by the voltage sources, which compensates for the extra unknowns caused by their (initially) unknown currents,

$$v_5 - v_4 = U \quad (6)$$

$$v_3 - v_2 = ku_x. \quad (7)$$

One of those equations introduced a further unknown, u_x, which reminds us that we need to define the marked (but unknown) quantities controlling any dependent sources in the circuit:

$$u_x = v_5. \quad (8)$$

Now there are 8 equations, but 9 unknowns. The opamp is guilty of having introduced an unknown output current i_o *and* an unknown output voltage v_1 that is not directly defined in the way that it would be for a normal dependent voltage source. But the usual opamp assumption (negative feedback and an ideal opamp) lets us state that the opamp input potentials must be equal,

$$v_3 = v_5. \quad (9)$$

End. That's it. The above 9 equations in 9 unknowns should be solvable, to find all node potentials and the unknown currents.

We can check the above equations by solving them in Matlab, and comparing this with a circuit solution in Spice.

```
s = solve( ...
  '0 = v1/R1 + (v1-v2)/R2 - io', ...
  '0 = (v2-v1)/R2 - ibeta - Isrc', ...
  '0 = ibeta + (v3-v4)/R3', ...
  '0 = (v4-v3)/R3 - ialpha', ...
  '0 = v5/R4 + ialpha', ...
  'v5 - v4 = Usrc', ...
  'v3 - v2 = k * ux', ...
  'ux = v5', ...
  'v3 = v5', ...
  'v1,v2,v3,v4,v5,ialpha,ibeta,io,ux' )  % solve for these variables

% look at the solutions for node potentials, in the output structure 's':
s.v1      (R2*Usrc + R4*Usrc - Isrc*R2*R3 - R4*Usrc*k)/R3
s.v2      (R4*Usrc - R4*Usrc*k)/R3
s.v3      (R4*Usrc)/R3
s.v4     -(Usrc*(R3 - R4))/R3
s.v5      (R4*Usrc)/R3
```

Then some numbers can be substituted, to allow comparison with the results from a circuit-solver. Here, we make a second structure, `n`, with fields containing the numeric values of fields in `s`:

```
Usrc=3, Isrc=0.1, k=7, R1=47, R2=68, R3=22, R4=12
clear n;
for f=fields(s)', n.(f{1}) = subs(s.(f{1})); end
  n =
    ialpha: -0.1364
     ibeta: -0.1364
        io: -0.1199
        ux:  1.6364
        v1: -7.3455
        v2: -9.8182
        v3:  1.6364
        v4: -1.3636
        v5:  1.6364
```

It's nice to double-check that there wasn't an error in writing those equations. The following SPICE netlist describes the same circuit. The component called `E1` is a VCVS that models the opamp as having a 'high' gain, $A = 10^9$. You can solve such models online at NGspice: just put the text into the first big text-box, then do `Simulate and Plot`, then inspect the `Raw output file` (a link below the plot); the plot is useless for our dc circuit. If you're actually interested in the syntax, see this good SpiceOverview.

```
EI1120_VT16_KS1Q2
V1    5 4  DC   3
I1    0 2  DC   0.1
E1    1 0  5 3  1e9
E2    3 2  5 0  7
R1    1 0      47
R2    1 2      68
R3    3 4      22
R4    5 0      12
.OP
.PRINT DC V(0) V(1) V(2) V(3) V(4) V(5)
.END
```

Spice returns the same values for node potentials as we obtained above:

```
v1:  -7.3455    v2:  -9.8182    v3:   1.6364    v4:  -1.3636   v5:   1.6364
```

---

3)

a) The value of I to obtain maximum power *into* the current-source is:

$$I = \frac{-3U}{2R}.$$

The common thing to do in such a case is to find the Thevenin (or Norton) equivalent that the component I 'sees' at its terminals. Then we know that the maximum power is obtained when the terminal voltage of that equivalent is half of its open circuit value; at this point, the output current is half of the short-circuit value.

The circuit simplifies to three identical branches in parallel with the current source. Each branch has a series U and R. Notice that the resistor R_4 is not relevant: it is in parallel with U_3, so it does not affect what the current source sees.

The Thevenin equivalent is then a voltage U and resistance $R/3$. Find this by nodal analysis (one KCL equation), or source transformation, or even superposition.

The short-circuit current is thus $U/(R/3)$ in the downward direction between the two terminals. Half of this – the maximum power condition for the circuit that the current source is connected to – is $\frac{1}{2}i_{\text{sc}} = U/(R/3)/2$. As the current I is defined upwards, we need a negative sign.

b) The power into the current source, when the above condition is fulfilled, is:

$$P_{I,\text{absorbed}} = \frac{3U^2}{4R}.$$

This is the product of half the short-circuit current and half the open-circuit voltage.

It could be done directly, without thinking of an equivalent or a maximum-power rule, by writing one KCL, solving for power as a function of I, then differentiating to find the value of I that gives maximum power. (The equation for power is quadratic, and we're told in the question that there's a point of maximum power transfer *to* the source I. So we can be lazy and not bother checking whether we've found a maximum or minimum.)