KTH EI1120 Elkretsanalys (CENMI), Omtenta 2016-06-09 kl 14–19

Tentan har 9 tal i 3 delar: tre tal i del A (12p), två i del B (10p) och fyra i del C (18p).

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text eller diagram; stor eller liten textstorlek, ... Det behöver *inte* lämnas in.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner av variabler.

Tips: Dela tiden mellan talen. *Senare deltal brukar vara svårare* att tjäna poäng på: fastna inte på dessa. Det hjälper, ofta, att rita om ett diagram för olika tillstånd eller med ersättningar eller borttagning av delar som inte är relevanta för det sökta värdet. Då blir kretsen ofta mycket lättare att tänka på och lösa. Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Räknande av betyg: Låt A, B och C vara de maximala möjliga poängen från delarna A, B och C i tentan, d.v.s. A=12, B=10, C=18. Låt a, b och c vara poängen man får i dessa respektive delar i tentan, och a_k vara poängen man fick från kontrollskrivning KS1, och b_k poängen från KS2, och h bonuspoängen från hemuppgifterna. Godkänd tentamen (och därigenom hel kurs) kräver:

$$\frac{\max(a,a_\mathrm{k})}{A} \geq 0{,}4 \quad \& \quad \frac{\max(b,b_\mathrm{k})}{B} \geq 0{,}4 \quad \& \quad \frac{c}{C} \geq 0{,}3 \quad \& \quad \frac{\max(a,a_\mathrm{k}) + \max(b,b_\mathrm{k}) + c + h}{A+B+C} \geq 0{,}5.$$

Betyget räknas också från summan över alla delar och bonuspoäng, d.v.s. sista termen ovan, med gränser (%) av 50 (E), 60 (D), 70 (C), 80 (B), 90 (A). Om tentan blev underkänd med liten marginal, så kan betyget Fx registreras, med möjlighet att få betyget E om ett kompletteringsarbete är godkänt inom några veckor efter tentamen.

Nathaniel Taylor (073 949 8572)

Del A. Likström

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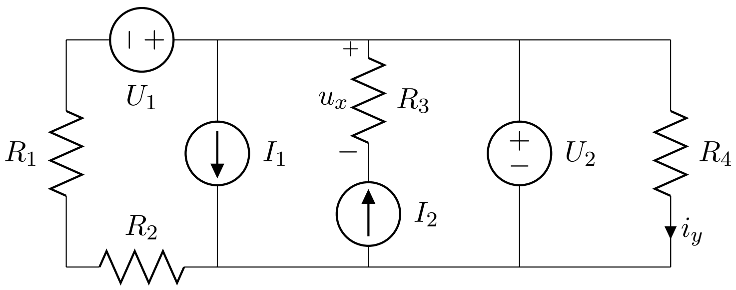
1) [4p] Bestäm de följande storheterna:

a) [1p] spänningen u_x

b) [1p] strömmen i_y

c) [1p] effekten levererad av källan I_1

d) [1p] effekten levererad av källan U_2



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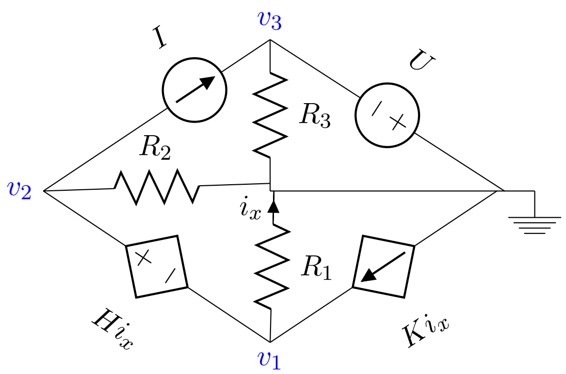
2) [4p]

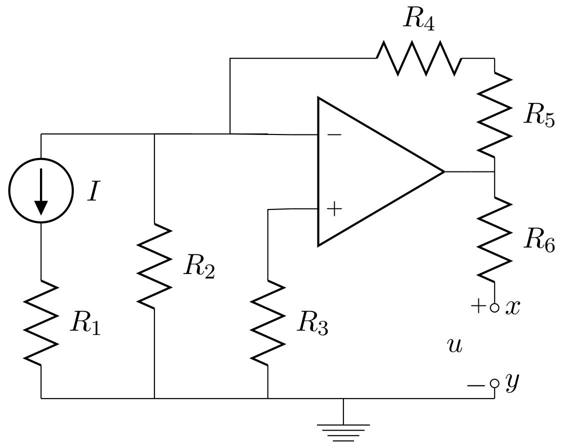
Använd nodanalys för att skriva ekvationer som skulle kunna lösas för att få ut de markerade nodpotentialerna v_1, v_2, v_3.

Du behöver bara visa att du kan översätta från kretsen till ekvationerna: du *måste inte* lösa eller förenkla ekvationerna.



3) [4p]

Alla motstånd i kretsen ($R_{1\cdots6}$) har resistans R. Vissa motstånd spelar ingen roll för lösningen. Slutlösningarna ska uttryckas bara med storheterna I och R, och förenklas.

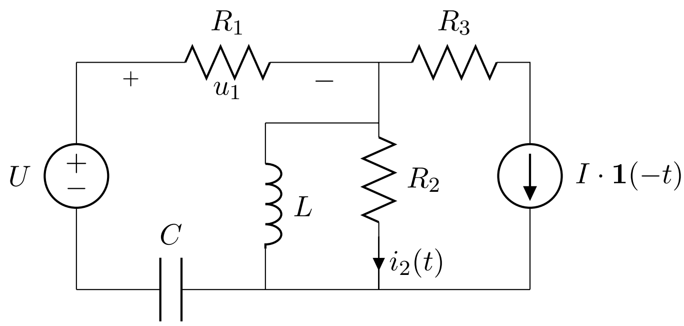


a) [3p] Bestäm Theveninekvivalenten av kretsen, med avseende på polerna 'x' och 'y'. Rita upp ekvivalenten, med dessa poler markerade.

b) [1p] Ett sjunde motstånd, R_7, kopplas mellan polerna x-y. Vilket värde måste det har för att den maximala möjliga effekten ska försörjas från kretsen till motståndet?

Del B. Transient

4) [5p] Bestäm följande fem storheterna:

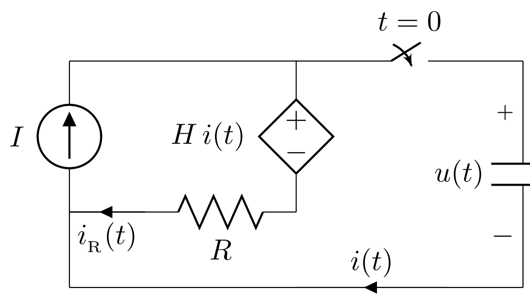


a) [2p] $u_1(0^+)$ och $i_2(0^+)$.

b) [2p] $u_1(\infty)$ och $i_2(\infty)$.

c) [1p] energin i kondensatorn vid $t \to \infty$.

5) [5p]



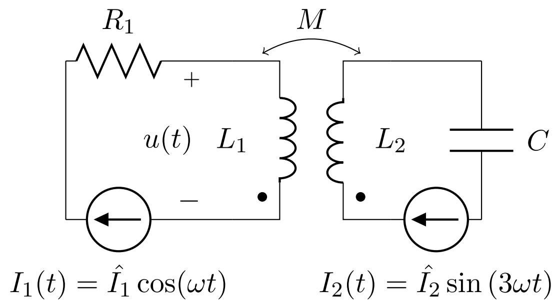
a) [4p] Bestäm $u(t)$, för $t > 0$.

b) [1p] Bestäm $i_R(t)$, för $t > 0$.

Begynnelsevärdet $u(0) = 0$ kan antas.

Del C. Växelström

6) [5p]



Spolarna är kopplade, med ömsesediginduktans M.

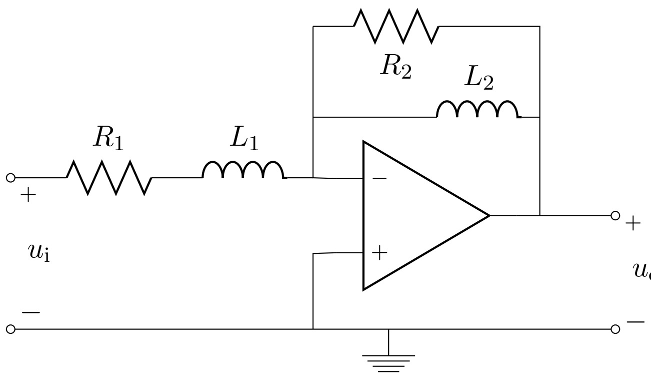
a) [4p] Bestäm $u(t)$.

b) [1p] Vilken aktiv effekt levereras av källan I_1?

7) [5p]

a) [2p] Bestäm kretsens nätverkfunktion,

$$H(\omega) = \frac{u_\mathrm{o}(\omega)}{u_\mathrm{i}(\omega)}.$$



b) [1p] Visa att svaret till deltal 'a' kan skrivas i den följande formen,

$$H(\omega) = \frac{-\mathrm{j}\omega/\omega_1}{(1+\mathrm{j}\omega/\omega_2)(1+\mathrm{j}\omega/\omega_3)}.$$

c) [2p] Skissa ett Bode amplituddiagram av funktionen $H(\omega)$ från deltal 'b'. Anta att $\omega_1 \ll \omega_2$, och att $\omega_2 \ll \omega_3$. Markera viktiga punkter och lutningar.

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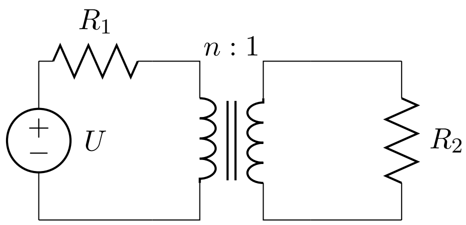
8) [3p]

Värmeelementen R_2 matas genom en lång, tunn ledning R_1 från en växelspänningkälla med effektivvärde U och vinkelfrekvens ω, genom en transformator som har kvoten $n:1$. Vilken effekt utvecklas i elementen R_2?



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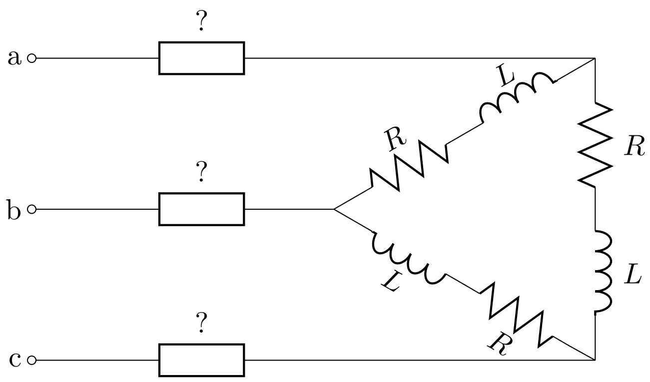
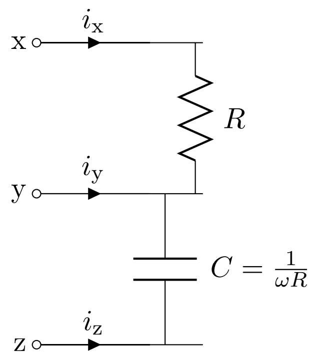
9) [5p]

Polerna a,b,c och x,y,z i kretsarna nedan visar anslutningar till trefas spänningskällor. Båda källor har vinkelfrekvens ω och huvudspänning U (därför blir t.ex. $|u_\mathrm{ab}| = U$ och $|u_\mathrm{xy}| = U$).

Som vanligt kan antas: källorna är ideala och balanserade, och effektivvärdeskala används.





a) [3p] I den vänstra kretsen, vilken komponent (typ och värde) ska användas där '?' är markerad, om effektfaktorn sett från källan (vid polerna a,b,c) ska vara 1?

b) [1p] I den högra kretsen, vilken ström (i_x, i_y, i_z) har lägste magnitud?
Fasföljden kan antas vara x,y,z; d.v.s. $v_\mathrm{y} = v_\mathrm{x}\,\mathrm{e}^{-\mathrm{j}2\pi/3}$.

c) [1p] Bestäm magnituden (absolutvärdet) av strömmen som du vald i deltal 'b'.

Slut. *Men slösa inte eventuell återstående tid: kolla och dubbelkolla svaren.*

Solutions (EI1120, VT16, 2016-06-09)

Q1.

a) $u_x = -I_2 R_3$

The series connection of I_2 and R_3 determines the current in R_3 (by KCL), then the voltage across this resistor is determined by Ohm's law.
Be careful about the directions: the usual form of Ohm's law, $u = iR$, assumes the 'passive convention', i.e. that the directions of u and i are defined with i entering the resistor at the end where u is marked positive.

b) $i_y = U_2/R_4$

The parallel connection of U_2 and R_4 determines the voltage across R_4 (by KVL), then the current through this resistor is determined by Ohm's law.

c) $P_{\text{I1}} = -I_1 U_2$

The power delivered *by* a current source can be found by determining the voltage across the source, and multiplying this by the source's current: the voltage should be defined with its positive reference (+-mark) where the current comes out of the source, else a negative sign is needed.
In this case, the source I_1 is directly in parallel with U_2, although it might not look it: try KVL around the loop of these two components. But the positive reference of U_2 is connected to the side of the current source where I_1 is defined as going *in*; the minus sign is needed to give the requestion direction of the power (power delivered by source I_1).

d) $P_{\text{U2}} = U_2 \left(\frac{U_2}{R_4} + \frac{U_2 - U_1}{R_1 + R_2} + I_1 - I_2 \right)$

This is a similar principle to part c, except that here we need to find the current delivered by the voltage source U_2, out from its positive-marked terminal. This is actually quite easily found, by KCL in the node above or below the source. There are four parallel branches in which this current can flow. Two of these have currents determined by current sources. We already know i_y. The current in R_1, R_2 is easy if the right loop is chosen for KVL, going round U_2, U_1, R_1, R_2. (Remember, we shouldn't assume the voltage across a current source is zero!)

Q2.

Extended nodal analysis ("the simple way")

Let's define the unknown currents in the voltage sources, with the positive direction going into the source's + terminal: i_α in the independent voltage source U, and i_β in the dependent voltage source $H i_x$.

Write KCL (let's take outgoing currents) at all nodes except ground:

$$\begin{aligned} \text{KCL}(1): \quad 0 &= \frac{v_1}{R_1} - i_\beta - K i_x && (1) \\ \text{KCL}(2): \quad 0 &= \frac{v_2}{R_2} + i_\beta + I && (2) \\ \text{KCL}(3): \quad 0 &= \frac{v_3}{R_3} - I - i_\alpha . && (3) \end{aligned}$$

The voltage sources introduced the problem of two extra unknowns in the above equations; they can solve this problem by providing two extra equations without further unknowns:

$$\begin{aligned} v_3 &= -U && (4) \\ v_2 - v_1 &= H\, i_x . && (5) \end{aligned}$$

The controlling variables of the dependent sources need to be defined in terms of the other known or unknown quantities. The controlling variable of both dependent sources is the current i_x, marked in R_1; this is

$$i_x = \frac{v_1}{R_1}. \tag{6}$$

The above is a sufficient set of equations for a solution.

Q3.

a) $U_{\mathrm{T}} = 2RI$ and $R_{\mathrm{T}} = R$.
The Thevenin voltage is the voltage u in conditions of open-circuit at x-y. In open-circuit, this is the same as the opamp's output voltage, as no current passes in R_6 in this situation. Similarly, there is no voltage across R_3, as no current flows in the opamp's inputs: so the non-inverting input is held to zero potential. We assume that the negative feedback on an ideal opamp causes the two inputs to have equal potential, which means there is also zero voltage at the inverting input. By Ohm's law again, no current flows in R_2. From KCL at the node of the inverting input, the current I must flow through R_5 and R_4, as R_2 and the inverting input have zero current.
The opamp's output is like an ideal voltage source: it is whatever value is needed in order to hold the inverting input to zero, through the feedback; the output current at x-y does not influence the feedback circuit. So the voltage u is simply $2RI - iR$, where the iR term is the output current i (x-y) and the resistance R of component R_6.

b) $R_7 = R_{\mathrm{T}} = R$. It's a classic dc maximum-power case: make the load equal to the source resistance.

Q4.

a) $u_1(0^+) = \frac{-IR_1R_2}{R_1+R_2}$, and $i_2(0^+) = \frac{IR_1}{R_1+R_2}$.

b) $u_1(\infty) = 0$, and $i_2(\infty) = 0$.

c) Energy in capacitor C as $t \to \infty$: $\frac{1}{2}CU^2$.

Q5.

a) $u(t) = IR\left(1 - \mathrm{e}^{-\frac{t}{(R-H)C}}\right) \qquad (t > 0)$.

This is probably easiest by writing an equation that relates $u(t)$ and $i(t)$ without the capacitor present. A Thevenin equivalent can then easily be obtained, from which the time constant and final value are found.
By KCL, $i_{\mathrm{R}}(t) = I - i(t)$.
By KVL, $u(t) = Hi(t) + Ri_{\mathrm{R}}(t)$.
Thus, $u(t) = Hi(t) + R(I - i(t)) = IR + (H - R)i(t)$.

Compare this to the equation for a Thevenin source, with the same definition directions of u and i: $u = U_{\mathrm{T}} - R_{\mathrm{T}}i$. This implies that the capacitor (after $t = 0$) sees a source with Thevenin voltage IR and Thevenin resistance $R - H$. The time-constant is thus $C(R - H)$, and the final value of $u(t)$ is IR.

Alternatively a differential equation can be obtained by substituting $i(t)$ using the capacitor's relation $i(t) = C\frac{\mathrm{d}u(t)}{\mathrm{d}t}$ into the earlier expression for $u(t)$, to get it entirely in terms of $u(t)$ and $\frac{\mathrm{d}u(t)}{\mathrm{d}t}$ instead of $u(t)$ and $i(t)$.

b) $i_{\mathrm{R}}(t) = I - C\frac{\mathrm{d}u(t)}{\mathrm{d}t} = I\left(1 - \frac{R}{R-H}\,\mathrm{e}^{-\frac{t}{(R-H)C}}\right) \qquad (t > 0)$.
The current in the capacitor is found from $i(t) = C\frac{\mathrm{d}u(t)}{\mathrm{d}t}$, then the current $i_{\mathrm{R}}(t)$ is found from KCL.

Q6.

[An error in the sign of the second term of Q6a solution has lain undiscovered for 9 months: thank you for pointing it out! The answer is now made more thorough, too.]

a) The solution of part 'a' is made easier by the fact that each of the two mutually-coupled inductors is in series with a current source. The equation describing two coupled inductors, in the time-domain, is $u_x(t) = L_x \frac{\mathrm{d}}{\mathrm{d}t} i_x(t) + M \frac{\mathrm{d}}{\mathrm{d}t} i_y(t)$ to find the voltage induced in inductor x due to currents in both inductors x and y. We've more usually seen this equation in its frequency-domain form, as $u_x(\omega) = \mathrm{j}\omega L_x i_x(\omega) + \mathrm{j}\omega M i_y(\omega)$. We must of course be careful about signs: we must check how the current directions we've defined in each inductor are related to the voltages and to the dots.

If the currents in both of the coupled inductors are known, then we can immediately write the equation for the voltage on one of the inductors, independently of the equation for the voltage on the other inductor. (It is more difficult if one or both of the currents is not determined at the start, but must be solved based on the mutual-inductor equations for the voltages on *both* inductors *and* the equations for the things connected to each side. If you want an example like that, try replacing both current sources with voltage sources, which gives two mutual inductance equations and two KVL equations to solve!)

Method 1.

In our lucky case with both currents determined, we could actually solve for $u(t)$ directly in the time-domain, perhaps more easily than using ac analysis (jω-metoden), by combining the mutual inductance equations with what we studied in the first 'transients' topic. Let's define currents $i_1(t)$ and $i_2(t)$ in the two inductors, both going in at the 'top', so they both come out at the dots. Then i_1 and the marked u follow the passive convention; with this, and the same direction relative to the dots, we can write the equation for $u(t)$ with no negative signs needed, and then identify the currents in terms of the sources in the circuit,

$$u(t) \;=\; L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t} + M \frac{\mathrm{d}i_2(t)}{\mathrm{d}t} \;=\; L_1 \frac{\mathrm{d}I_1(t)}{\mathrm{d}t} - M \frac{\mathrm{d}I_2(t)}{\mathrm{d}t}$$

into which we substitute the given functions for the current sources, and differentiate,

$$u(t) \;=\; L_1 \frac{\mathrm{d}}{\mathrm{d}t} \hat{I}_1 \cos(\omega t) - M \frac{\mathrm{d}}{\mathrm{d}t} \hat{I}_2 \sin(3\omega t) \;=\; -\omega L_1 \hat{I}_1 \sin(\omega t) - 3\omega M \hat{I}_2 \cos(3\omega t).$$

There was no need of defining a 'u_2' across the other inductor.

With a slightly more difficult circuit, such as with voltage sources or with the resistor in parallel instead of in series with L_1, this direct use of differentiation would not be so straightforward, because the currents through the inductors wouldn't be immediately known. Then it would become actually desirable to do ac analysis instead of using a time-domain short-cut.

Method 2.

Now we will show the ac way of considering the original question, to get the same answer as above.

We want to find the voltage u, across inductor L_1. There are two independent sources in the circuit. They are sinusoidal, so we can use ac analysis for the steady-state behaviour of the circuit. They have different frequencies so we can't use ac analysis for both at the same time: a "phasor solution" is about specifying amplitudes and phases (angles) of sinusoidal quantities *at one specific frequency*. However, we can use superposition to solve for the case where sources at one frequency are active and the others are zero, and then vice versa. In our case, a zero source (zero current-source) means no current in the respective series-connected inductor, which makes the solution quick and convenient.

SUPERPOSITION STATE 1: I_1 active, I_2 set to zero.
The frequency is ω. Let's take a cosine reference. We have therefore a source $I_1(\omega) = \hat{I}_1 \underline{/0}$, putting current into the top of the coil L_1. At the other side, source I_2 is zero, i.e. an open circuit, so no current flows in coil L_2.

Based on the frequency-domain equation for the voltage on one of two mutually coupled inductors,

$$u_{(1)}(\omega) \;=\; \mathrm{j}\omega L_1 I_1(\omega) + \mathrm{j}\omega M \cdot 0 \;=\mathrm{j}\omega L_1 \hat{I}_1,$$

and after converting this back to a time-function, we get

$$u_{(1)}(t) \;=\; \omega L_1 \hat{I}_1 \cos(\omega t + \pi/2) \;=\; -\omega L_1 \hat{I}_1 \sin(\omega t).$$

SUPERPOSITION STATE 2: I_1 set to zero, I_2 active.
The frequency is 3ω. There's no need to use the same reference as above – we have to do the whole time→frequency→time conversion separately for each frequency. So let's choose a sine reference. We have then a source $I_2(\omega) = \hat{I}_2\underline{/0}$, putting current into the *bottom* of the coil L_2; I prefer to see this as a source $-I_2(\omega) = \hat{I}_2\underline{/\pi}$ putting current into the *top* of that coil, so that the mutual inductor equations don't need any negative sign. At the other side, source I_1 is zero, i.e. an open circuit, so no current flows in coil L_1.

Using the same equation as before,

$$u_{(2)}(\omega) \;=\; \mathrm{j}\omega L_1 \cdot 0 \;+\; \mathrm{j}3\omega M \hat{I}_2\underline{/\pi} \;=\; 3\omega M \hat{I}_2\underline{/\tfrac{3\pi}{2}}$$

where the subscript '(2)' shows that it's the part of u due to the superposition state 2 (*not* that it's the voltage on the second inductor L_2). Converted into a time-function, this is

$$u_{(2)}(t) \;=\; 3\omega M \hat{I}_2 \sin\left(3\omega t + {}^{3\pi}\!/_2\right) \;=\; -3\omega M \hat{I}_2 \sin\left(3\omega t + {}^{\pi}\!/_2\right) \;=\; -3\omega M \hat{I}_2 \cos(3\omega t).$$

COMBINE THE SUPERPOSITION STATES.
The final step is to combine the superposition results, in the time-domain. (We reiterate: this cannot be done with the phasors, since we have two 'different species of phasors', i.e. at different frequencies; phasor calculations are based on the assumption of one frequency.)

$$u(t) \;=\; u_{(1)}(t) + u_{(2)}(t) \;=\; -\omega L_1 \hat{I}_1 \sin(\omega t) - 3\omega M \hat{I}_2 \cos(3\omega t).$$

b) The active power delivered by source I_1 is: $\frac{1}{2}\, I_1^2 R_1$.

The long way of finding this is to find the complete voltage across the source $I_1(t)$, and thus to find the source's power. Let's call this voltage $U_1(t)$, defined according to the active convention, i.e. the '+'-side is where the current $I_1(t)$ is defined as coming out.

Then, by KVL in the left loop,

$$U_1(t) \;=\; R_1 I_1(t) + u(t) \;=\; R_1 \hat{I}_1 \cos(\omega t) - \omega L_1 \hat{I}_1 \sin(\omega t) - 3\omega M \hat{I}_2 \cos(3\omega t),$$

which can be written all in terms of cos as

$$U_1(t) \;=\; R_1 \hat{I}_1 \cos(\omega t) + \omega L_1 \hat{I}_1 \cos\left(\omega t + {}^{\pi}\!/_2\right) - 3\omega M \hat{I}_2 \cos(3\omega t).$$

The instantaneous power (the actual power-output at each instant, as a time-function) is simply the product of $U_1(t)$ and $I_1(t)$,

$$P(t) \;=\; \hat{I}_1 \cos(\omega t)\, \Big(R_1 \hat{I}_1 \cos(\omega t) + \omega L_1 \hat{I}_1 \cos\left(\omega t + {}^{\pi}\!/_2\right) - 3\omega M \hat{I}_2 \cos(3\omega t) \Big)$$

If you expand out all the cos terms, using the relation $\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha{+}\beta) + \cos(\alpha{-}\beta)\right]$, you will get six terms, of which only one is a constant; the rest are all sinusoidal, and therefore have a mean value of zero. The constant term is $\frac{1}{2}\hat{I}_1^{\,2} R_1$, which gives the power that's actually delivered over significant numbers of periods, ignoring the 'oscillating' power over time. This mean value is what active power in ac circuits describes. Using the $\cos\alpha \cdot \cos\beta$ relation, we see that if $\cos(\omega t)$ is multiplied by $\cos(n\omega t + \phi)$, there is only a non-zero mean value if $n = 1$ and $\phi \neq \pm\pi/2$.

The ac way to approach this power question is to talk about the "power superposition principle". What we have normally seen in ac power questions is that all independent sources, and therefore all other quantities in a linear circuit, are sinusoidal at *one* frequency. In our present question, however, there are sources with different frequencies. As shown above, the voltage across one of the current sources is a sum of two freqencies. Power superposition says that the power (active power) into or out of a component is the sum of the powers due to each frequency in the circuit acting alone. Notice that this is *not* true for superposition of powers due to sources with the *same* frequency, as can easily be verified for a simple case like two current sources and a resistor all in parallel. It's only true for the different frequencies, because all terms that involve a product of sinusoids (i.e. current and voltage across a component) at *different* frequencies will have a zero mean and thus not contribute to the active power transfer.

Using the power-superposition approach we consider the power from source 1 due to each source separately. With I_2 set to zero, the second inductor L_2 can be ignored; the circuit is just the source I_1, in series with a resistor R_1 and inductor L_1, so the source delivers the power that is used by the resistor $\frac{1}{2}\hat{I_1}^2 R_1$, since no active power is used in an inductor. The factor $1/2$ is due to $\hat{I_1}$ being a peak value. With I_1 set to zero, there is no current in this source (!) so it cannot be consuming or providing any active or reactive power.

Notice a careful silence about reactive power. This is not a well defined quantity in the way that active power is (related to average rate of energy transfer); it can be, and is, defined in lots of ways for different purposes. We've stayed mainly with complex power in single-frequency circuits with two-terminal situations; then reactive power is pretty well defined. Start adding further wires through which the power flow is to be defined, or having more than one frequency, and different opinions will arise.

Q7.

a) This is a classic case of an inverting amplifier, except that each of the two main impedances (input, and feedback) consists of two components.

For an inverting amplifier with input impedance Z_1 and feedback impedance Z_2, the sought relation is $\frac{u_o}{u_i} = -\frac{Z_2}{Z_1}$, which can be found from KCL at the node of the inverting input, with the standard opamp-with-negative-feedback assumption that this node is at the same potential as the non-inverting input (zero).

Replacing the impedance symbols with the component values (series for the input, parallel for the feedback),

$$\frac{u_o}{u_i} = -\frac{\frac{j\omega L_2 R_2}{R_2 + j\omega L_2}}{R_1 + j\omega L_1} = \frac{-j\omega L_2 R_2}{(R_1 + j\omega L_1)(R_2 + j\omega L_2)}$$

At this point, you might have chosen to make further manipulations to get the expression closer to what we show below for part 'b'.

b) Continuing the manipulation from part 'a',

$$\frac{u_o}{u_i} = \frac{-j\omega L_2 R_2}{(R_1 + j\omega L_1)(R_2 + j\omega L_2)} = \frac{-j\omega L_2/R_1}{(1 + j\omega L_1/R_1)(1 + j\omega L_2/R_2)}.$$

This final expression becomes the requested form, if we substitute $\omega_1 = R_1/L_2$, $\omega_2 = R_1/L_1$, $\omega_3 = R_2/L_2$; note that the definitions for ω_2 and ω_3 could be swapped, without making the solution wrong, as both denominator terms have similar form.

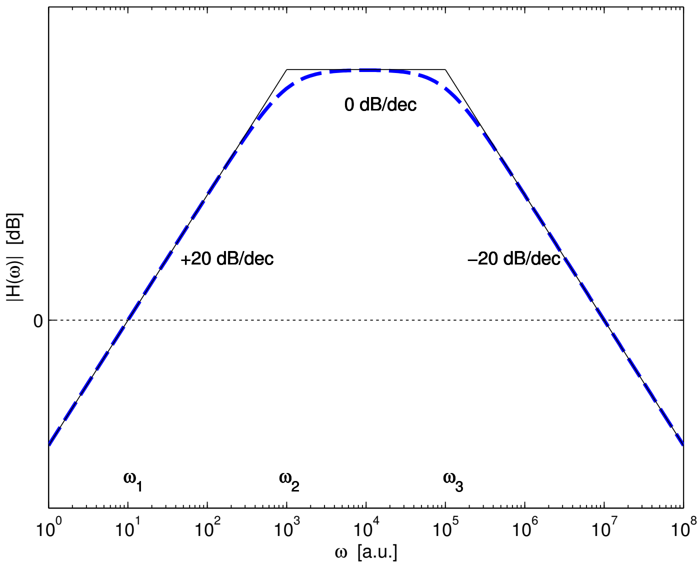
c) Sketch a Bode amplitude plot of $H(\omega)$, assuming $\omega_1 \ll \omega_2$ and $\omega_2 \ll \omega_3$.

This is the same as in the 2016-03 EI1120 exam, except that here the relation between ω_1 and ω_2 is not specifically defined as 100, but just as a large difference.

The Bode amplitude plot is shown on the right: the frequency is in arbitrary units, and the ratios ω_2/ω_1 and ω_3/ω_2 have been chosen as 100.

The main features that should be marked are the 0 dB point at $\omega = \omega_1$, the changes of gradient at ω_2 and ω_3, and the gradients of ± 20 dB/decade. Including the 0 dB/decade gradient in the pass-band is nice but not necessary, as it's obvious for a flat line!

The classic asymptotic Bode amplitude plot has just the straight lines; the further curve shows the actual function plotted numerically.



Q8.

$$P_{\mathrm{R2}} = n^2 R_2 \left(\frac{U}{R_1 + n^2 R_2}\right)^2.$$

Several methods can be chosen here. Probably the easiest is to replace R_2 and the ideal transformer with a simple model that shows how these two components "look" to the rest of the circuit. In other words, what does the transformer (with R_2 connected to its secondary) look like at its primary terminals? Because an ideal transformer has no losses, the power going into the transformer's primary is the same as the power into R_2, so if we find this power we have our solution directly.

A resistor R seen from the primary of a $N_1 : N_2$ transformer appears as $(N_1/N_2)^2 R$. In our case, the primary of the transformer appears as a load of $n^2 R_2$. The current around the primary circuit is $\frac{U}{R_1+n^2R_2}$; by the expression for power dissipation, $i^2 R$, the above solution for the power can be obtained.

Q9.

a) '?' $= C = \frac{3}{\omega^2 L}$.
It probably simplifies the thinking if we convert this balanced 3-phase Δ-load (R,L) to an equivalent Y load ($R/3$,$L/3$). Then, unity power-factor (PF $= 1$) can be achieved by making the unknown component cancel the inductor. It should therefore be a capacitor, with value $\frac{1}{\mathrm{j}\omega C} = -\mathrm{j}\omega L/3$.

b) Current i_{y} has the lowest magnitude. This may seem strange, as line y carries the currents of *both* loads (R and C) whereas the other lines carry just one or the other current. The current *magnitudes* in these two components are identical, due to the given relation $C = \frac{1}{\omega R}$. The reason why the sum is less than either of the separate values is that there is a large phase-shift, quite close to 180°, between these currents.
The voltage between lines y-x has a 60° displacement compared to the voltage between lines y-z: think of the triangle of line-voltages. The current in the capacitor has a 90° phase shift (leading) compared to its voltage, whereas the current in the resistor is in phase with its voltage. The result is that these two currents, of identical magnitude, have a 150° phase-shift, giving a sum that is less than the individual values. If the capacitor and resistor were swapped, the current in line y would be almost twice the current in each of the other lines, as the 60° and 90° shifts would then partially cancel instead of adding.

c) $|i_y| = \frac{U}{R}\sqrt{2-\sqrt{3}}$
As described above, the two branches (R and C) supplied by line y have currents that have equal mag-

nitude but a 150° phase-shift. The phasor addition to find the magnitude of two phasors with magnitude 1 and relative angle 150° can be done numerically as `abs( 1+exp(1j*150*pi/180) ) =  0.52`; this means the current in line y is about half the magnitude of the currents in lines x and z. To calculate it analytically, we can define one phasor to be purely real, and the other to be split into real and imaginary parts corresponding to cos and sin of 150°,

$$|1+1\underline{/150°}| = |1+\cos 150° + \mathrm{j}\sin 150°| = \left|1-\frac{\sqrt{3}}{2}+\mathrm{j}\frac{1}{2}\right| = \sqrt{\left(1-\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{2-\sqrt{3}}.$$

Multiplying this factor by the current magnitude in each component, U/R, the above solution for i_{y} is obtained.