KTH EI1102 / EI1100 Elkretsanalys Omtenta 2016-06-09 kl 14–19

Tentan har 6 tal i 2 delar: tre tal i del A (15p), tre i del B (15p).

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text eller diagram; stor eller liten textstorlek. Det behöver *inte* lämnas in.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner av variabler.

Tips: Dela tiden mellan talen. Senare deltal brukar vara svårare att tjäna poäng på: fastna inte på dessa.
Det hjälper, ofta, att rita om ett diagram för olika tillstånd eller med ersättningar eller borttagning av delar som inte är relevanta för det sökta värdet. Då blir kretsen ofta mycket lättare att tänka på och lösa. Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Godkänd tenta kräver minst 25% i del A, 25% i del B, och 50% i genomsnitt (båda delar). Betyget räknas från summan över båda delar, med gränser (%) av 50 (E), 60 (D), 70 (C), 80 (B), 90 (A).

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Del A. Likström och Transient

1) [5p]

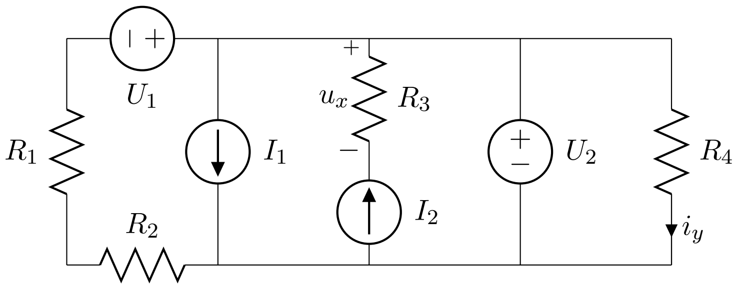
Bestäm de följande storheterna:

a) [1p] spänningen u_x

b) [1p] strömmen i_y

c) [1p] effekten levererad av källan I_1

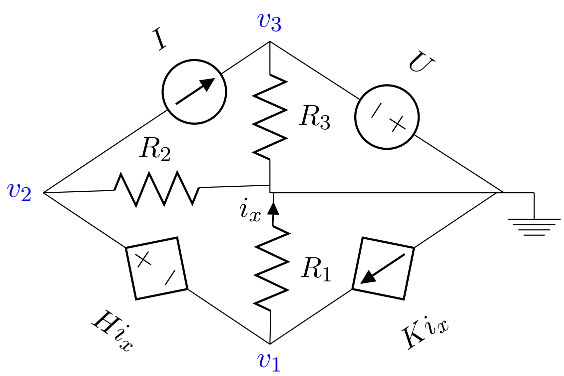
d) [2p] effekten levererad av källan U_2



2) [5p]

Använd nodanalys för att skriva ekvationer som skulle kunna lösas för att få ut de markerade nodpotentialerna v_1, v_2, v_3.

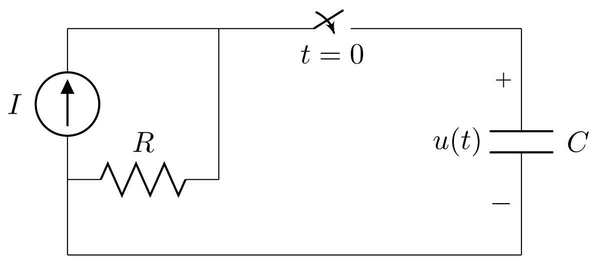
Du behöver bara visa att du kan översätta från kretsen till ekvationerna: du *måste inte* lösa eller förenkla ekvationerna.



3) [5p]

Bestäm $u(t)$, för $t > 0$.

Begynnelsevärdet $u(0) = 0$ kan antas.

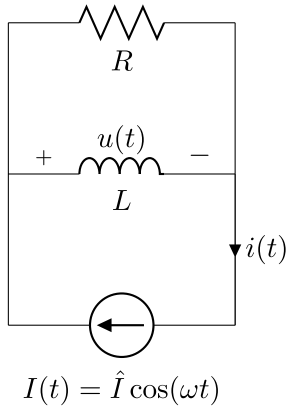


Del B. Växelström

4) [5p]

a) [4p] Bestäm $u(t)$.

b) [1p] Bestäm $i(t)$.



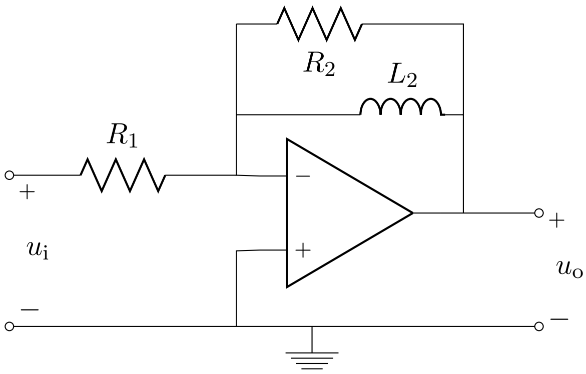
5) [5p]

a) [2p] Bestäm kretsens nätverkfunktion,

$$H(\omega) = \frac{u_{\mathrm{o}}(\omega)}{u_{\mathrm{i}}(\omega)}.$$

b) [1p] Visa att svaret till deltal 'a' kan skrivas i den följande formen,

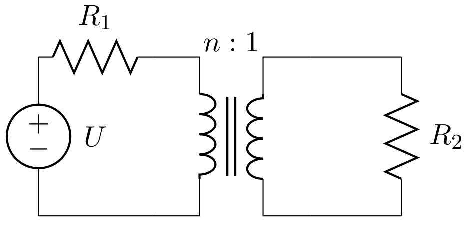
$$H(\omega) = \frac{-\mathrm{j}\omega/\omega_1}{(1+\mathrm{j}\omega/\omega_2)}.$$



c) [2p] Skissa ett Bode amplituddiagram av funktionen $H(\omega)$ från deltal 'b'. Anta att $\frac{\omega_2}{\omega_1} = 1000$. Markera viktiga punkter och lutningar.

6) [5p]

Värmeelementen R_2 matas genom en lång, tunn ledning R_1 från en växelspänningkälla med effektivvärde U och vinkelfrekvens ω, genom en transformator som har kvoten $n : 1$. Vilken effekt utvecklas i elementen R_2?



Solutions (EI1102/EI1100, VT16, 2016-06-09)

Questions 1, 2 and 6 are the same as questions in the simultaneous re-exam for the CENMI program. More detailed solutions of these can be found here: `2016-06_EM_omtenta.pdf`

Q1.

a) $u_x = -I_2 R_3$

b) $i_y = U_2/R_4$

c) $P_{\mathrm{I1}} = -I_1 U_2$

d) $P_{\mathrm{U2}} = U_2 \left(\frac{U_2}{R_4} + \frac{U_2 - U_1}{R_1 + R_2} + I_1 - I_2 \right)$

Q2.

We'll show Extended nodal analysis. Let's define the unknown currents in the voltage sources, with the positive direction going into the source's + terminal: i_α in the independent voltage source U, and i_β in the dependent voltage source Hi_x.

Write KCL (let's take outgoing currents) at all nodes except ground:

$$\begin{aligned}
\mathrm{KCL}(1): \quad 0 &= \frac{v_1}{R_1} - i_\beta - K i_x & (1)\\
\mathrm{KCL}(2): \quad 0 &= \frac{v_2}{R_2} + i_\beta + I & (2)\\
\mathrm{KCL}(3): \quad 0 &= \frac{v_3}{R_3} - I - i_\alpha & (3)\\
v_3 &= -U & (4)\\
v_2 - v_1 &= H\, i_x & (5)\\
i_x &= \frac{v_1}{R_1}. & (6)
\end{aligned}$$

The above is a sufficient set of equations for a solution.
Many other choices are possible.

Q3.

We're told that, at $t = 0$, the capacitor has no charge: $u(0) = 0$. At that time, it's connected by a switch to a Norton source, of I and R. The final state is $u(\infty) = IR$, with the capacitor behaving as an open circuit so that all the source current passes through the resistor. The time-constant is $\tau = RC$. Using these initial and final values and the time-constant, the relation $y(t) = y(\infty) + (y(0) - y(\infty))\, \mathrm{e}^{-t/\tau}$ gives

$$u(t) = IR \left(1 - \mathrm{e}^{-t/CR}\right) \qquad (t > 0).$$

The solution could instead be derived by finding and solving the differential equation of the circuit. This solution can be found as an example in the course material for the time-functions Topic within transient analysis: the circuit is a classic case of a discharged capacitor being connected to a Norton source.

Q4.

a) This is in the ac part of the exam, so sinusoidal steady-state conditions (stationärt tillstånd) can be assumed. We'll use ac analysis ("jω-metoden"). The source can be represented as a phasor: if we use peak value and cosine-reference, this gives $I(\omega) = \hat{I}$. The inductor is an impedance $\mathrm{j}\omega L$, and the resistor is an impedance R. Then $u(\omega)$ can be found as the voltage across the combined parallel impedance when a current $I(\omega)$ passes through it:

$$u(\omega) = I(\omega) \frac{\mathrm{j}\omega L R}{R + \mathrm{j}\omega L} = \frac{\mathrm{j}\omega L R \hat{I}}{R + \mathrm{j}\omega L}.$$

For converting back to a time-function it is convenient to express the above result in polar form:

$$u(\omega) = \frac{\omega L R \hat{I}}{\sqrt{R^2 + \omega^2 L^2}} \angle \tfrac{\pi}{2} - \tan^{-1} \tfrac{\omega L}{R}.$$

Now, being careful to use the same choice of peak value and cosine reference, this can be written in the time-domain,

$$u(t) = \frac{\omega L R \hat{I}}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t + \frac{\pi}{2} - \tan^{-1} \frac{\omega L}{R}\right).$$

b) This is supposed to be easy: a check of confidence in basic circuit concepts. By KCL the marked current must be the source's current: they're in series. So,

$$i(t) = \hat{I} \cos(\omega t).$$

Q5.

a) This is a classic case of an inverting amplifier, but with a feedback impedance that consists of two components in parallel.

For an inverting amplifier with input impedance Z_1 and feedback impedance Z_2, the sought relation is $\frac{u_o}{u_i} = -\frac{Z_2}{Z_1}$, which can be found from KCL at the node of the inverting input, using the standard opamp-with-negative-feedback assumption that this node is at the same potential as the non-inverting input (zero).

Replacing the impedance symbols with the component values (series for the input, parallel for the feedback),

$$\frac{u_o}{u_i} = -\frac{\frac{j\omega L_2 R_2}{R_2 + j\omega L_2}}{R_1} = \frac{-j\omega L_2 R_2}{R_1\ (R_2 + j\omega L_2)}$$

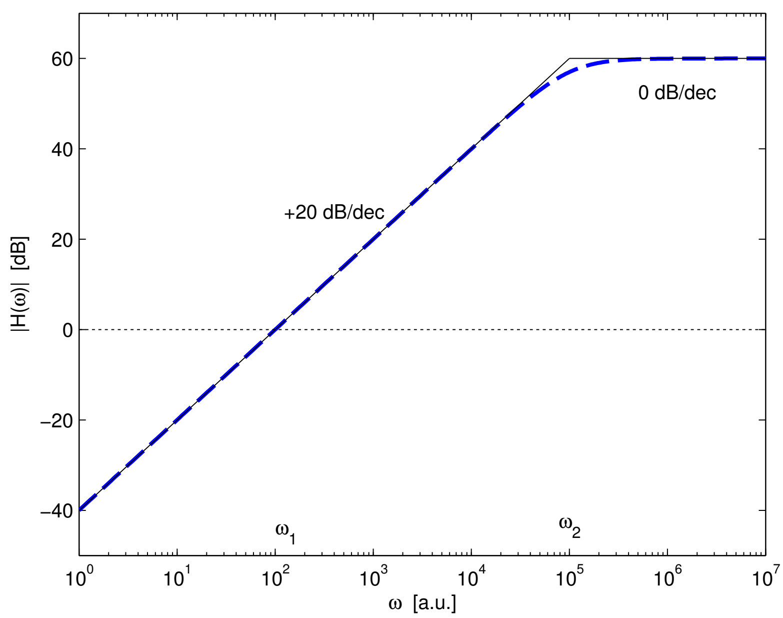
At this point, you might have chosen to make further manipulations to get the expression closer to what we show below for part 'b'.

b) Continuing the manipulation from part 'a',

$$\frac{u_o}{u_i} = \frac{-j\omega L_2 R_2}{R_1\ (R_2 + j\omega L_2)} = \frac{-j\omega L_2 / R_1}{1 + j\omega L_2 / R_2}.$$

This final expression becomes the requested form, if we substitute $\omega_1 = R_1/L_2$ and $\omega_2 = R_2/L_2$.

c) The Bode amplitude plot is shown below: the frequency is in arbitrary units. Notice that the negative sign in the expression has no effect, as this is a *magnitude* plot. The classic asymptotic Bode amplitude plot has just the straight lines; the further curve shows the actual function plotted numerically.



The main features that should be marked are the 0 dB point at $\omega = \omega_1$, the gradient of +20 dB/decade at the low-frequency side of the plot, and the change of gradient at $\omega = \omega_2$. Explictly marking the 0 dB/decade gradient at the high-frequency side is not required, as long as the plot clearly shows a horizontal line. Marking of the +20 dB/decade gradient can also be omitted, as long as the axis markings (gain and frequency) make clear what the gradient is. The question gives enough detail that the 60 dB level of the flat part of the plot can be deduced.

Q6.

$$P_{\mathrm{R2}} = n^2 R_2 \left(\frac{U}{R_1 + n^2 R_2} \right)^2 .$$

Hint: a resistor R seen from the primary of a $N_1 : N_2$ transformer appears as $(N_1/N_2)^2 R$. Alternatively, a solution can be found by defining currents and starting from KVL equations on the two sides.