KTH EI1110 Elkretsanalys (CELTE) KS 1 — 2016-09-27 kl 08–10

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text eller diagram; stor eller liten textstorlek, ... Det behöver *inte* lämnas in.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner av variabler.

KS1 ger inte direkt betyg, utan poäng som kan ersätta poängen i sektion-A i tentan (TEN1, oktober) om KS:en gav mer. Se därför reglerna för TEN1 angående kraven.

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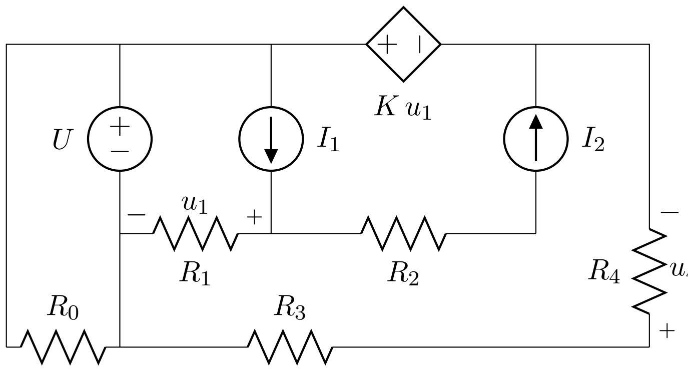
1) [4p] Bestäm de följande storheterna:

a) [1p] effekten absorberad av R_2

b) [1p] effekten absorberad av R_0

c) [1p] spänningen u_1

d) [1p] spänningen u_4



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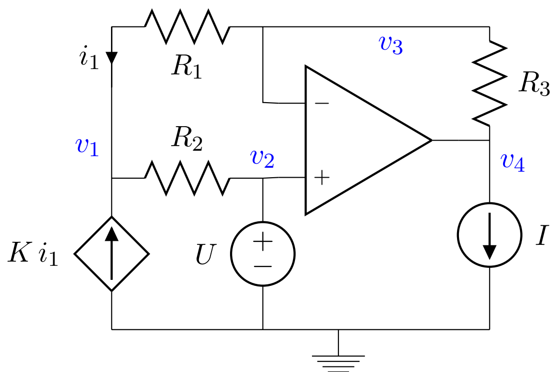
2) [4p]

Använd nodanalys för att skriva ekvationer som skulle kunna lösas för att få ut de markerade potentialerna v_1, v_2, v_3, v_4.

Du behöver bara visa att du kan översätta från kretsen till ekvationerna: du *måste inte* lösa eller förenkla ekvationerna.



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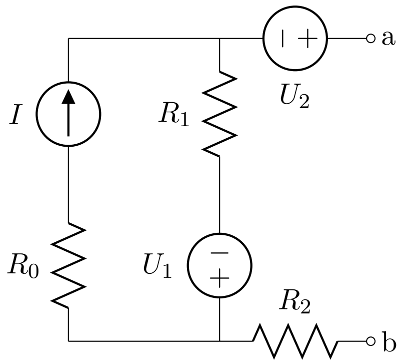
3) [4p]

a) [3p] Bestäm Theveninekvivalenten av kretsen, med avseende på polerna 'a' och 'b'. Rita upp ekvivalenten, med dessa poler markerade.

b) [1p] En annan komponent ansluts mellan a-b. Bestäm strömmen som ska passera genom komponenten från pol 'a' till pol 'b' om komponenten ska få den maximala möjliga effekten från den ritade kretsen.



Slut. *Men slösa inte eventuell återstående tid: kolla och dubbelkolla svaren.*

Solutions (EI1110 KS 1 HT16, 2016-09-27)

Q1.

a) $\boxed{P_{\text{R2,in}} = I_2^2 R_2}$
KCL: series connection of R_2 and I_2 determines the current in R_2.

b) $\boxed{P_{\text{R0,in}} = U^2/R_0}$
find voltage on R_0 from parallel connection of U and R_0, i.e. KVL around leftmost loop.

c) $\boxed{u_1 = (I_1 - I_2)\, R_1}$
KCL at the right of R_1, then Ohm's law in R_1.

d) $\boxed{u_4 = \dfrac{KR_1R_4\,(I_1 - I_2) - UR_4}{R_3 + R_4}}$
Resistors R_3 and R_4 are in series. We can apply KVL in a loop consisting of these resistors, independent source U and dependent source Ku_1. If we define i_4 as the current going upwards in R_4, then $U - Ku_1 + i_4R_4 + i_4R_3 = 0$, from which $i_4 = (Ku_1 - U)/(R_3 + R_4)$. From Ohm's law, $u_4 = R_4i_4 = (Ku_1 - U)R_4/(R_3 + R_4)$. Voltage division could have been used instead to get here. As u_1 is only a 'marked quantity' (and we assume only component values are known quantities by default), we should replace this with its value found in part 'c'.

Q2.

Extended nodal analysis ("the simple way")

Let's define the unknown currents in the voltage sources: i_α into the + terminal of the independent voltage source U, and i_β out of the opamp output.

Write KCL (let's take outgoing currents) at all nodes except ground:

$$\text{KCL}(1): \quad 0 = -Ki_1 + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} \tag{1}$$

$$\text{KCL}(2): \quad 0 = \frac{v_2 - v_1}{R_2} + i_\alpha \tag{2}$$

$$\text{KCL}(3): \quad 0 = \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_4}{R_3} \tag{3}$$

$$\text{KCL}(4): \quad 0 = -i_\beta + \frac{v_4 - v_3}{R_3} + I \tag{4}$$

The voltage source introduced the trouble of an extra unknown, i_α, in the above equations; it also provides the solution of this 'trouble' by providing an equation that doesn't contain further unknowns:

$$v_2 = U \tag{5}$$

The opamp similarly introduced an extra unknown, i_β. Its output voltage is not directly known in the same way the the source's voltage is. We can consider the opamp output as a dependent voltage source (other side connected to earth). With a 'non-ideal' opamp of finite gain, we could say that $v_4 = (\text{gain}) \cdot (v_2 - v_3)$, just as we would with a voltage-dependent voltage source whose controlling variable is the opamp's differential input $(v_+ - v_-)$. With the assumption of infinite gain, we consequently assume that the differential input is forced to zero. Using the dependent-source model, the output then ends up as $\infty \cdot 0$, which isn't directly helpful: we find it easier if instead we ignore the output potential and include the equation relating the input potentials,

$$v_2 = v_3 \tag{6}$$

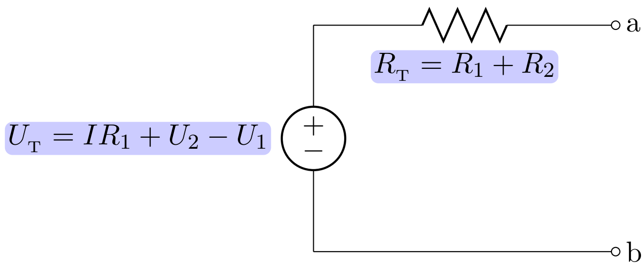
The controlling variables of the dependent sources need to be defined in terms of the other known or unknown quantities. The controlling variable of both dependent sources is the current i_1, marked in R_1; this is

$$i_1 = \frac{v_3 - v_1}{R_1}. \tag{7}$$

The above 7 equations are sufficient for a solution.

Q3.

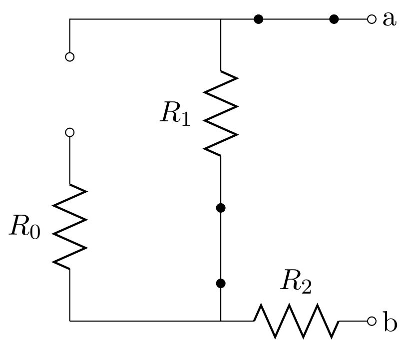
a) The Thevenin equivalent can be drawn as the following:



To find the open-circuit voltage (equal to Thevenin voltage) notice that if a-b are open-circuit then there is no voltage across R_2, and all the current I must pass downwards through resistor R_1; then KVL around R_2, U_1, R_1, U_2 and a-b gives the open-circuit voltage $u_{ab(oc)}$ of 'a' relative to 'b'

$$0 \cdot R_2 - U_1 + IR_1 + U_2 - u_{ab(oc)} = 0.$$

To find the Thevenin resistance, the short-circuit current (Norton current) could be found, e.g. by nodal analysis (one node, three branches) or source transformation or superposition: then the resistance can be found as the ratio of Thevenin voltage to Norton current. But in this circuit with just independent sources it is probably easier to find the equivalent resistance directly by setting the sources to zero and combining the remaining resistors.



b) $i_{ab(maxP)} = \dfrac{IR_1 + U_2 - U_1}{2(R_1 + R_2)}$.

Maximum power transfer from the source circuit (or its Thevenin equivalent) is obtained at half its short-circuit current. The short circuit current is U_T/R_T passing externally from 'a' to 'b': so the current for maximum power is $U_T/2R_T$.