# KTH EI1120 Elkretsanalys (CENMI) KS 1   2017-02-03 kl 08–10

**Hjälpmedel:** Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text eller diagram; stor eller liten textstorlek, . . . . Det behöver *inte* lämnas in.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. $R$ för ett motstånd, $U$ för en spänningskälla, $k$ för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas. Var tydlig med diagram och definitioner av variabler.

KS1 ger inte direkt betyg, utan poäng som kan ersätta poängen i sektion-A i tentan (TEN1, mars) om KS:en gav mer. Se därför reglerna för TEN1 angående gränser.

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**1)** [4p]

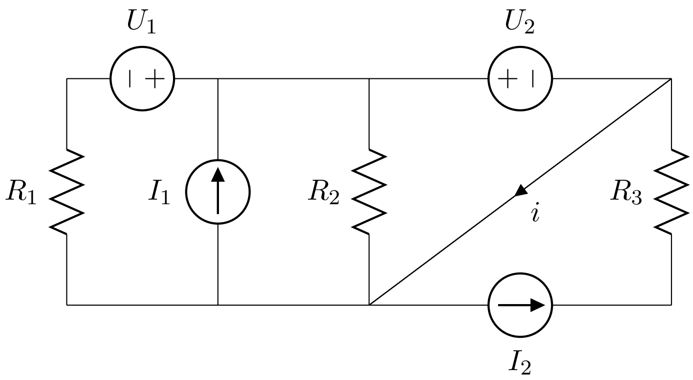
Bestäm följande storheter:

**a)** [1p] effekten absorberad av $R_3$

**b)** [1p] effekten absorberad av $R_1$

**c)** [1p] effekten levererad av källan $U_2$

**d)** [1p] strömmen $i$

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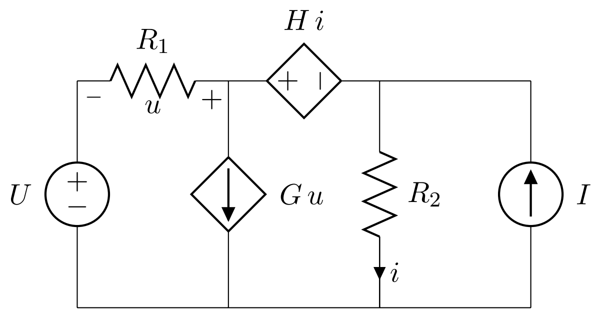
**2)** [4p]

Bestäm den markerade spänningen $u$.

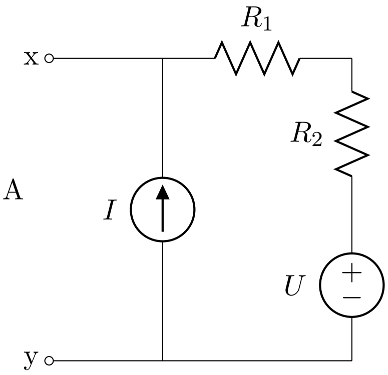
*Tips:* nodanalys, förmodligen bäst med supernodmetoden.



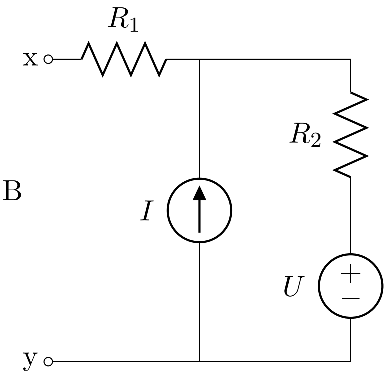
**3)** [4p]

Dessa två kretsar innehåller samma komponenter, men ihopkopplade på olika sätt. Komponenterna markeras med enskilda beteckningar för att förenkla hur lösningarna beskrivs, men här gäller det att $R_1 = R_2 = R$ och $U = IR$, där $R$ och $I$ är givna storheter. Därför ska $R_1$, $R_2$ och $U$ *inte* användas i slutsvaren till deltal 'a' och 'b'.

Krets A




Krets B




**a)** [1p] Vilket värde ska ett motstånd ha om det kopplas mellan polerna x-y för att få ut den maximala möjliga effekten från krets A?

**b)** [3p] Bestäm kvoten

$$\frac{P_{\max(A)}}{P_{\max(B)}}$$

där $P_{\max(A)}$ och $P_{\max(B)}$ är de maximala effekterna som kretsarna A respektivt B kan leverera.

## **Slut.** *Men slösa inte eventuell återstående tid: kolla och dubbelkolla svaren.*

Var inte besvikna att opamps, av en händelse, inte kom upp denna gång.
De brukar träffas senare under 'filter' ämnet i växelström.

**Short translations of the questions to English:**

1. Find:
a) the power absorbed by $R_3$
b) the power absorbed by $R_1$
c) the power delivered from source $U_2$
d) the marked current $i$ (d).

2. Find the marked voltage $u$.
Hint: nodal analysis is probably a good idea, using the supernode method.

3. Circuits A and B have the same components, with different connections. The components have unique names in order to describe them more neatly during a solution. However, $R_1 = R_2 = R$ and $U = IR$, where only $R$ and $I$ are the 'given' quantities. Thus, final answers should *not* contain $R_1$, $R_2$ or $U$ as the component values.
a) What value of resistance should be connected between terminals x-y in order to extract the maximum possible value of power from circuit A?
b) Find the ratio $P_{\max(A)}/P_{\max(B)}$, where the $P_{\max}$ values are the maximum power that the circuits A and B respectively can deliver.

# Solutions (EI1120 KS 1 VT17, 2017-02-03)

## Q1.

**a)** $P_{\mathrm{R3}} = I_2^2 R_3$
Series connection with current source determines this resistor's current (KCL).

**b)** $P_{\mathrm{R1}} = \frac{(U_1-U_2)^2}{R_1}$
KCL around the loop of $R_1, U_1, U_2$ gives a voltage $U_1 - U_2$ across the resistor; direction isn't important, as this term is squared when we calculate power in a resistor.

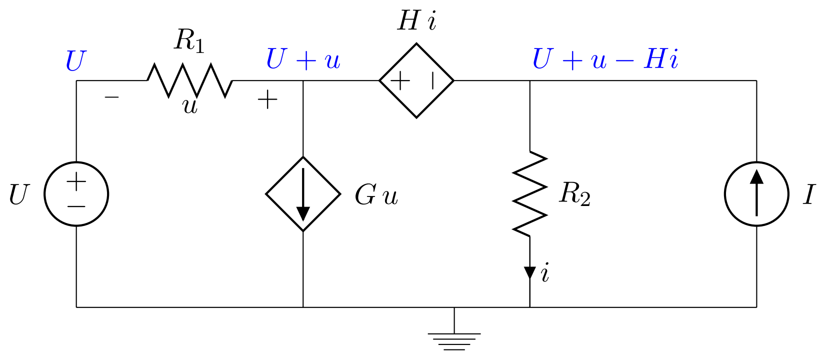
**c)** $P_{\mathrm{U2}} = U_2 \left( \frac{U_2}{R_2} + \frac{U_2-U_1}{R_1} - I_1 \right)$.
The product of voltage $U_2$, and the current coming out of the + terminal of this source, gives the power delivered from the source to the rest of the circuit, which is what is requested.
KCL at the node to the left of source $U_2$ gives the three terms for this current. The currents in the resistors are directly found from KVL then Ohm's law, because for each resistor there exists (in this circuit) a loop with only voltage sources and the resistor.

**d)** $i = I_1 + I_2 + \frac{U_1-U_2}{R_1} - \frac{U_2}{R_2}$.
This solution is found from KCL at the node to the left of source $I_2$.
The currents in the branches are found as in the previous subquestion, by KVL and Ohm's law in the resistors, and by the definition of the current sources.

## Q2.

You may have found a way that seems quicker than nodal analysis (well done if so), but it does seem that nodal analysis using the supernode method is likely to be a good, reliable way to get a solution without too many intermediate steps.

We already have $u$ defined, and this is what we want to find at the end. So let's use this in our definition of the potentials, after defining the bottom node to be the reference.




The current $i$ is, by Ohm's law, $i = \frac{U+u-Hi}{R_2}$.
Rearranging, $i = \frac{U+u}{R_2+H}$.

Now KCL can be written for the supernode:

$$\frac{U+u-U}{R_1} + G\,u + \frac{U+u}{R_2+H} - I = 0,$$

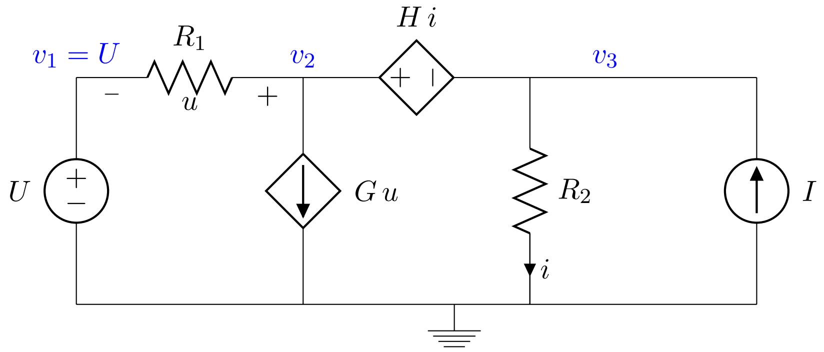
from which

$$u = \frac{I\,(R_2+H) - U}{1 + (R_2+H)\left(\frac{1}{R_1} + G\right)} \qquad \text{or} \qquad u = \frac{I - \frac{1}{R_2+H}U}{\frac{1}{R_2+H} + \frac{1}{R_1} + G}.$$

**The above is sufficient as a solution.**

We continue with some further comments, in case you want to think about doing supernode-based nodal analysis in a way closer to what we started with when we studied the subject. (In the above solution, we've been unconventional by expressing the KCL in terms of a marked voltage instead of node potentials. That seemed sensible, because that marked voltage was the only thing we had to find.)

The most formal way of doing this would be to define a reference (0 V), define supernodes and nodes, assign an unknown potential to each, then solve and subsequently find $u$ from the potentials. For example, we could make the following definitions.




If the bottom node is the reference, then the node $v_1$ becomes part of the 'ground supernode'; we already know its potential, and we don't need any KCL.

The remaining two nodes form a further supernode, joined by the dependent voltage source. We keep one of the potentials, e.g. we might choose potential $v_3$ in our case; then the other potentials in the supernode are defined in terms of this, which in our case means that $v_2 = v_3 + Hi$, which can be simplified to $v_2 = (1 + H/R_2)v_3$ by seeing that $i = v/R_2$.

Now KCL is needed for each supernode and each node not in a supernode, but not for the ground [super]node. To avoid the further unknown $u$ entering the KCL, we can substitute $u = (1+H/R_2)v - U$, so the current in the dependent current source is $G\left((1 + {}^{H}\!/_{R_2})\, v_3 - U\right)$.

After solving this for the unknown, $v_3$, other potentials can be found by working backwards: in our case, $v_2 = (1 + H/R_2)v_3$. Then our sought quantity – which was $u$ – can be found: it is $u = v_2 - U$.

**Q3.**

**a)** A resistance $2R$ will extract maximum power from terminals x-y of circuit A.
This choice of resistance follows from the maximum power theorem: for maximum power the resistance should equal the source resistance (Thevenin or Norton resistance) of the circuit it's connected to. The same is true for circuit B, as both of these circuits have the same Thevenin resistance, $R_1 + R_2$.
In these examples, which have no dependent sources, the Thevenin resistance is most conveniently found by setting all the sources to zero then simplifying the remaining resistors into a single resistor.

**b)** As mentioned in part 'a', both circuits have Thevenin resistance of $R_{\mathrm{T}} = R_1 + R_2 = 2R$.
By KVL, the open-circuit voltages (equal to the Thevenin voltages) of the circuits are:

$$U_{\mathrm{T(A)}} = U + I(R_1 + R_2) = 3IR \qquad\qquad U_{\mathrm{T(B)}} = U + IR_2 = 2IR.$$

The power delivered from a Thevenin source $U_{\mathrm{T}}$ & $R_{\mathrm{T}}$ to a resistor of $R_{\mathrm{T}}$ is $P = \frac{U_{\mathrm{T}}^2}{4R_{\mathrm{T}}}$.
The factor 4 appears because the voltage across the external resistor is half the Thevenin voltage (voltage division between equal resistors) and the power in a resistor is proportional to the square of voltage.

Thus, as we know that this choice of resistor the power gives the maximum power,

$$\frac{P_{\mathrm{max(A)}}}{P_{\mathrm{max(B)}}} = \frac{\frac{(3IR)^2}{4\cdot 2R}}{\frac{(2IR)^2}{4\cdot 2R}} = \left(\frac{3}{2}\right)^2 = 2.25.$$