KTH EI1120 Elkretsanalys (CENMI) KS 1 2019-02-01 kl. 08–10

Permitted material: Beyond writing-equipment, a single piece of paper up to A4 size can be brought, with free choice of content: handwritten, printed; small, large; text, diagram, image; one or both sides, etc. This paper does not need to be handed in with the exam.

Unless it is stated otherwise, the final answer to a question should be expressed in terms of the known quantities given in the question, and any clear simplifications should be done. Component values such as R for a resistor, U for an independent voltage source, or K for a dependent source, are assumed to be known quantities. Marked currents or voltages such as i_x are assumed to be definitions, not known quantities.

Clearly drawn and labelled diagrams are a good way to help yourself avoid mistakes, and to make clear to others what you are doing. By showing clearly your intermediate steps in a solution, you improve your chance of getting points even if the final result is wrong. You may write in Swedish or English; but we suggest that writing in either is seldom necessary if you make good use of diagrams and equations!

KS1 does not give any direct grade. Its points will be used to replace Section-A in the final exam or re-exam, if this would improve your points there. See therefore the rules for the exam to relate the points to grades: at least 40% is needed in Section-A alone, as well as 50% overall.

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1) [4p]

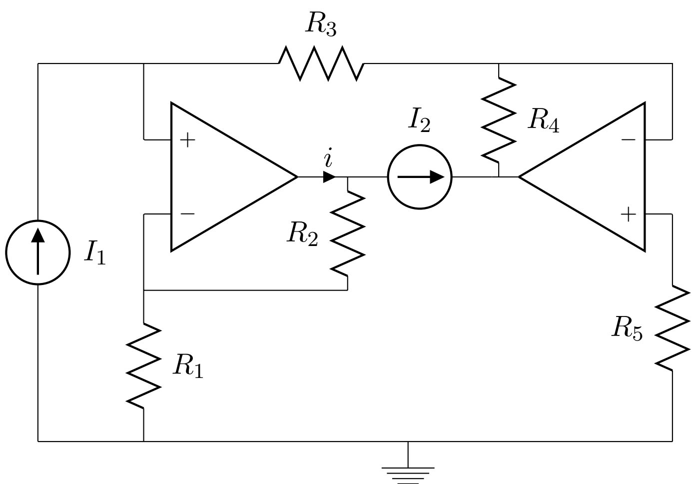
Find:

a) [1p] the power absorbed by R_5

b) [1p] the power absorbed by R_4

c) [1p] the marked current i

d) [1p] the power delivered from source I_2



Hint. This is possible by a step-by-step approach; but if you are stuck, try writing supernode-based equations and solving them. Two components' values do not affect the potentials.

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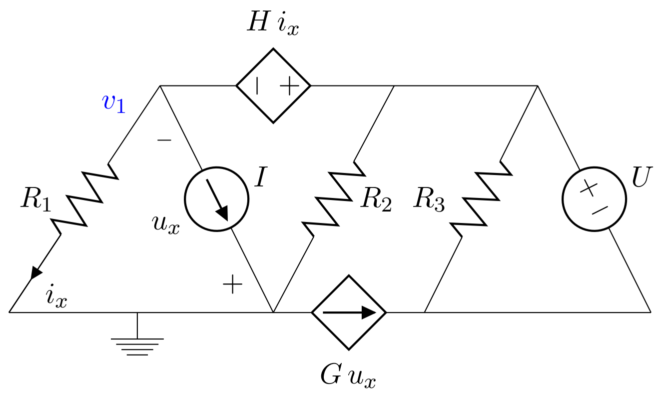
2) [4p]

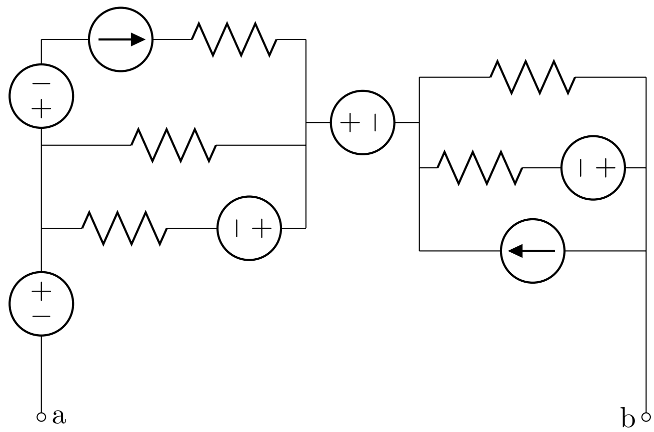
Determine potential v_1.

Suggestion: supernode method.

Alternative, for maximum 3.5p:
Write an equation or equations that could be solved for v_1. Your answer can contain further unknowns, but it must be possible to find v_1 in terms of known quantities (component values) from your answer without further information being needed.



3) [4p]



All resistors here have value R, all current-sources I, and all voltage-sources IR.

a) [3p] Show that the Thevenin equivalent of this circuit between terminals ‘a’ and ‘b’ is a resistance R and a voltage IR. You should include a diagram that shows the direction of the voltage relative to the terminals.

b) [1p] What is the highest power that this circuit can deliver at its terminals ‘a’ and ‘b’?

Översättningar:

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text, diagram, bild; stor eller liten textstorlek, o.s.v. Det måste inte lämnas in med skrivningarna.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, K för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas.

Var tydlig med diagram och definitioner av variabler. Du får skriva på svenska eller engelska, men vi rekommenderar att diagram och ekvationer används i stället i de flesta fall.

KS1 ger inte direkt betyg, utan poäng som kan ersätta poängen i sektion-A i tentan (TEN1, mars) om KS:en gav mer. Se därför reglerna för TEN1 angående gränser.

1. Bestäm följande storheter:

a) [1p] effekten absorberad av R_5.

b) [1p] effekten absorberad av R_4.

b) [1p] den markerade strömmen i.

d) [1p] effekten levererad från källan I_2.

2. Härled potentialen v_1.

Förslag: supernodmetoden.
Tips: det är förmodligen inte så svårt som det först ser ut.

Alternativt, för maximalt 3,5p: skriv en eller flera ekvationer som *skulle kunna* lösas för v_1. Dessa får innehålla andra okända variabler också, som du inte behöver eliminera. Ekvationerna måste räcka för lösning av v_1 utan vidare information.

3. Samtliga motstånden har värdet R, strömkällor värdet I, och spänningskällor värdet IR.

a) [3p] Visa att Theveninekvivalenten med avseende på polerna a-b är en spänning U och motstånd R. Rit även ett diagram som visar rätt riktning för spänningen.

b) [1p] Bestäm den högsta effekten som kan levereras från kretsen till någonting som är kopplad mellan ’a’ och ’b’.

The End. *Don’t waste remaining time . . . check your solutions!*

Solutions (EI1120 KS 1 VT19, 2019-02-01)

Q1.

a) Power into R_5 is 0.

No current flows into an ideal opamp's input, so by KCL there is no current through this resistor.

b) Power into R_4 is $I_1^2 R_4$.

Again, this is by KCL and the assumption of zero current in an opamp input. Applying KCL twice, at the left and the right of R_3, all the current from source I_1 is shown to pass through R_4.

c) The marked current: $i = I_2 + I_1 \frac{R_3}{R_1}$.

It may be a bit hard to find where to start. We want i, and see that this requires finding the current down in R_2 then using KCL. But this appears to need us to find the output potential of that left opamp ... how?

Start by looking anywhere for a 'weakness' of the problem – something we *can* state or easily calculate about the circuit. We already saw that R_5 carries no current, which by Ohm's law means there is no voltage across it: as one side of R_5 is connected to the reference node, the other side of it is also at zero potential. By the rule of an ideal opamp with negative feedback, the two inputs have the same potential, which tells us that the potential above R_4 must also be zero. The current I_1 passes through R_3, so by Ohm's law the upper input of the left opamp has potential $0 + I_1 R_3$.

Now we could go the quick way: by the opamp rule, the lower input of the left opamp has that same potential, which means $\frac{I_1 R_3}{R_1}$ passes down through R_1; then as no current flows in the opamp input, KCL tells us the current down R_2 is the same. A longer way is to note that the left opamp is connected as a classic non-inverting amplifier, so its output potential is this input potential multiplied by $\frac{R_1+R_2}{R_1}$, which is $\frac{I_1 R_3 (R_1+R_2)}{R_1}$. Because no current flows in the opamp input, the current down the series pair of resistors can be found by Ohm's law, $\frac{I_1 R_3 (R_1+R_2)/R_1}{R_1+R_2} = \frac{I_1 R_3}{R_1}$.

Then it is just necessary to do KCL at the node above R_2.

d) Power that source I_2 delivers to the circuit is $-I_1 I_2 R_4 - I_1 I_2 \frac{R_3(R_1+R_2)}{R_1}$.

The negative signs indicate that if the component values are positive, then the source is actually receiving power from the rest of the circuit. Resistors generally have positive values, unless they represent some strange electronic component instead of a simple piece of material. But the current source values might easily be negative. So we just do the algebra and accept that 'delivered by' or 'into' are *definitions* of which direction we're considering, not claims about where the power must actually be going.

To solve this, we need to find the voltage and current across the component in question, and multiply them in the right directions to find power out from the component, i.e. the positive product ui if i is defined *out* at the terminal where u is defined +). A current source has a specified current, so we just need to find the voltage across it.

The potential at the left side of source I_2 was already found in part 'c'. The potential at the right side can be found from KVL, knowing that a current I_1 passes downwards through R_4, and that the potential above R_4 is zero as shown in part 'c'. This gives an output potential of $-I_1 R_4$.

An alternative approach is to see the right opamp as a classic inverting amplifier, despite its being drawn in a non-classic way. Here, R_3 is the input, and R_4 is the feedback. This is not specially helpful, as we'd have to find the potential above the source I_1 in order to use the inverting-amplifier formula of $-R_4/R_3$, and doing that is about as much work as just finding the potential we were looking for, from direct analysis.

Now, knowing the potentials at both sides of source I_2, the power *delivered from* this source is

$$I_2 \left(-I_1 R_4 - \frac{I_1 R_3 \left(R_1 + R_2\right)}{R_1} \right).$$

Another way to start on this question is just to follow nodal analysis, preferably by the supernode method as we do not need to calculated the currents in the opamp outputs or in the voltage source. This way there is less need to think about the specific circuit, as long as one carefully applies the rules.

Q2.

The two voltage sources (one an independent source, one dependent) join three of the nodes, so these three can be treated as one supernode. The remaining node is the reference node, of potential 0. Therefore, only one unknown potential need be defined: we can use the v_1 that already is marked.

The potential to the right of the current-controlled voltage source is then $v_1 + Hi_x$.
We'll follow the principle of not including further unknowns in our equations, other than the accepted node-potentials; in this case, only v_1 is needed as we only have one KCL. So the marked but unknown quantity i_x should be written in terms of other quantities. It is defined as the current down R_1, which is $i_x = \frac{v_1-0}{R_1}$. We therefore write the potential not as $v_1 + Hi_x$ but as $v_1\left(1+\frac{H}{R_1}\right)$.

The potential at the further node, below the source U, is then $v_1\left(1+\frac{H}{R_1}\right) - U$.

The value of the dependent current source is Gu_x. The controlling quantity u_x is seen to be the voltage across the independent current source, such that $u_x = 0 - v_1$. Thus, the current in the dependent current source is $-Gv_1$.

Now, with the potentials and source values described in terms of just v_1, KCL at this supernode is:

$$0 + \frac{v_1}{R_1} + I + \frac{\left(1+\frac{H}{R_1}\right)v_1}{R_1R_2} - (-Gv_1) \; = \; 0$$

Notice in the above that currents within the supernode are not included, such as the current in R_3. (It doesn't hurt if you *do* include it, correctly, as it will then appear once positively and once negatively, cancelling.) Just the currents from this supernode to the other nodes (in this case there's only one) are included.

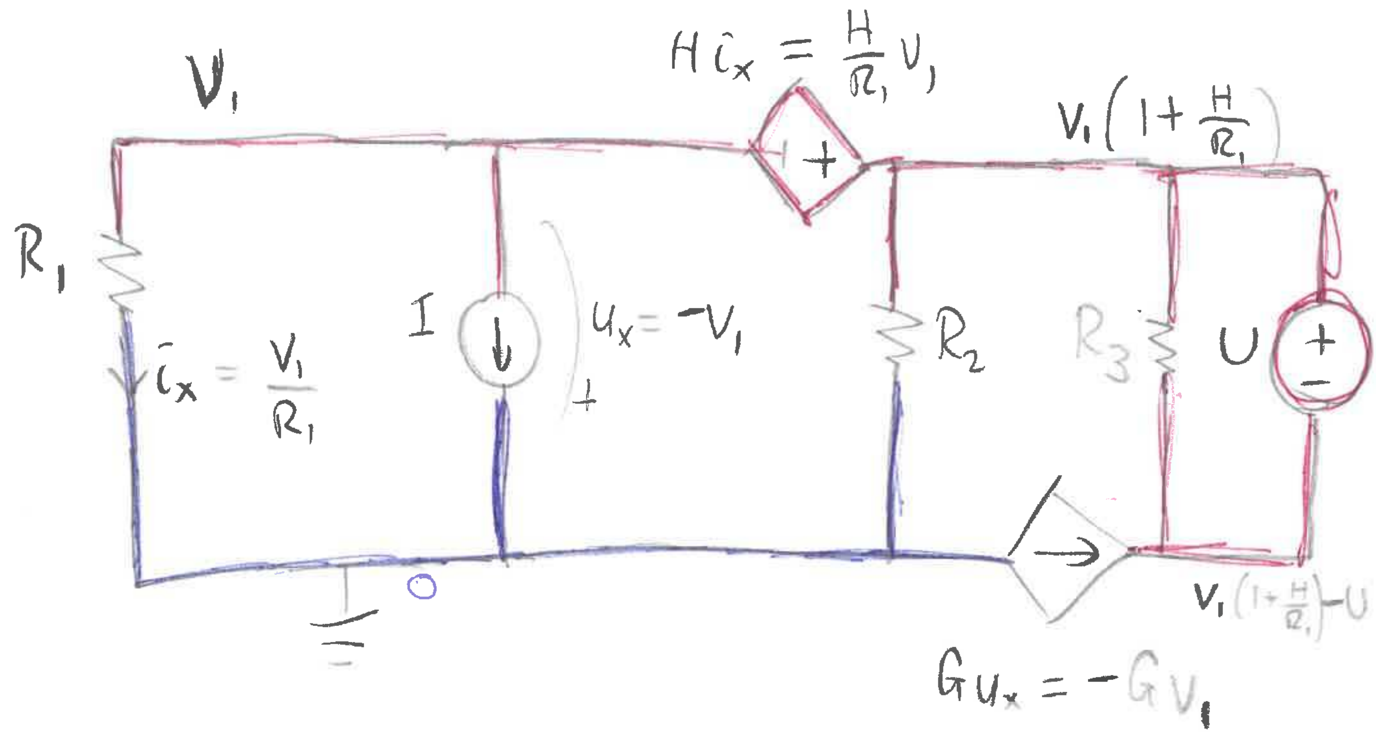
Rearranging the KCL equation for v_1,

$$v_1 \quad = \quad \frac{-I}{\frac{1}{R_1} + \frac{1+\frac{H}{R_1}}{R_2} + G} \quad = \quad \frac{-IR_1R_2}{R_1 + R_2 + H + R_1R_2G}.$$

A hand-written version of this solution is on the following page.

An opposite extreme of answer, for 'writing equations without solving them', could be three KCL equations (all nodes except reference), two for the voltage sources, and two for defining the controlling variables of the dependent sources.

② Supernode method: find v_1



4 nodes − 1 ground

3 others − all joined by voltage sources
⇒ all can be treated as one supernode:

define one potential, eg. V_1
then write others in terms of this

No KCL on ground node

KCL on the supernode (currents leaving the supernode)

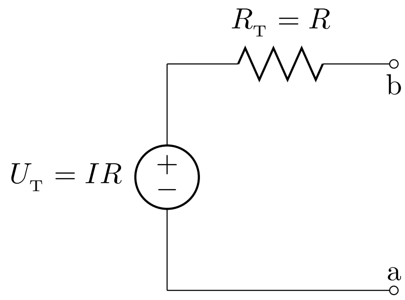
$$\frac{V_1}{R_1} + I + \frac{V_1\left(1+\frac{H}{R_1}\right) - 0}{R_2} - (-G V_1) = 0$$

$$V_1 = \frac{-I}{\frac{1}{R_1} + \frac{1+\frac{H}{R_1}}{R_2} + G} = \frac{-I}{\frac{R_2}{R_1 R_2} + \frac{R_1 + H}{R_1 R_2} + \frac{R_1 R_2 G}{R_1 R_2}}$$

$$\boxed{V_1 = \frac{-I R_1 R_2}{R_1 + R_2 + H + R_1 R_2 G}}$$

Q3.

a) The Thevenin equivalent is shown on the right. You could instead swap the source direction or sign, in order to write terminal ‘a’ at the top, which is more conventional.



The target solution is already given, so answers to this question should give clear working, to show they really proved the point independently!

Several methods can efficiently be applied. We have seen several when marking the 96 papers from this KS – thank you for the variety!

On the next page is a handwritten solution using repeated source-transformation. This is the method that was used when formulating the question.

Following that is a method that finds the circuit’s short-circuit current by nodal analysis, and the resistance by simplifying the resistors with all sources set to zero. When finding short-circuit current, the terminals are shorted so they can be seen as one node. By the supernode method, there are then just two main supernodes in the circuit, if we ignore the extra node in the top-left branch (this node isn’t important, as that whole branch could be replaced by just its current source). So one KCL finds the voltage between the two supernodes. From that, it is easy to calculate the current flowing at terminal ’a’ or terminal ’b’. When finding resistance by setting all sources to zero, it’s important to remember that if there were *dependent* sources these could not be set to zero; the method is mainly useful for cases such as this one, with just independent sources.

Another method we saw was a nice use of superposition in groups, taking all voltage sources (current sources set to zero) then vice versa. This worked neatly.

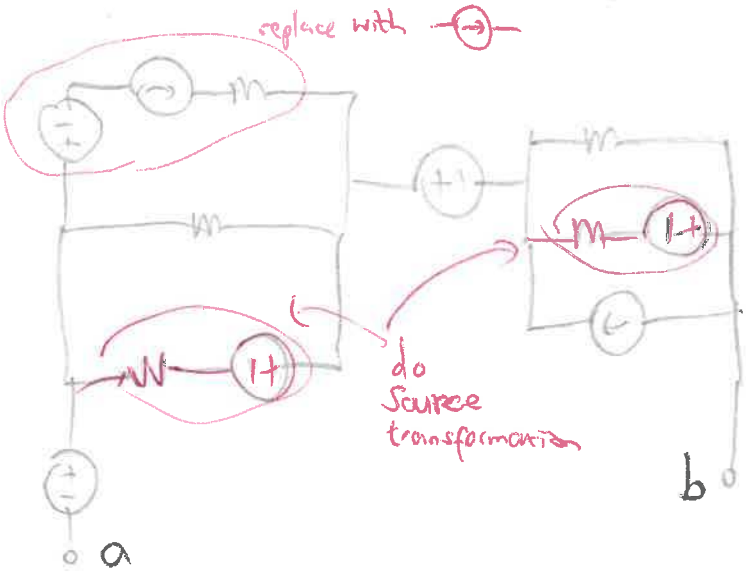
b)

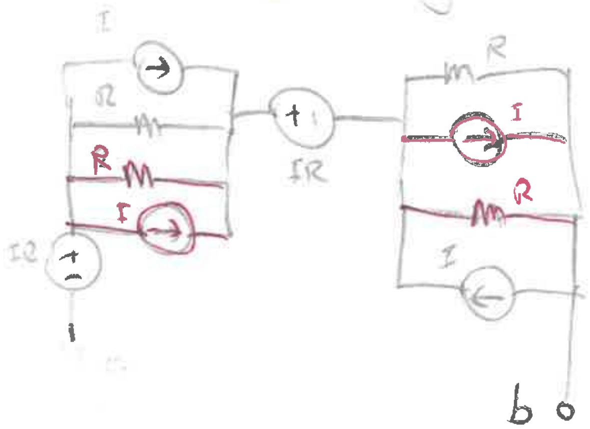
$$P_{\max} = \frac{1}{4} I^2 R$$

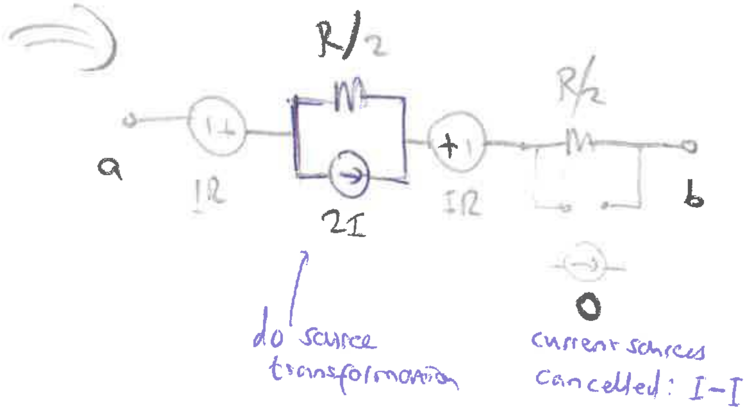
For reasoning, see definition/proof of maximum power output: a linear circuit (or its two-pole equivalent) will have half its short-circuit current, and half its open-circuit voltage, at the maximum power condition.

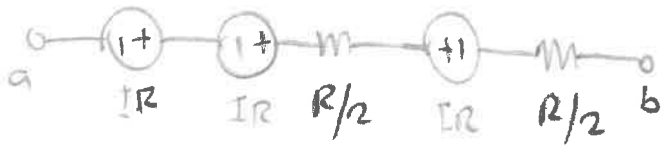
The solution is not $I^2 R$, although that came up several times in the submitted solutions. If the whole short-circuit current is extracted from the Thevenin source, there is no voltage at the terminals, so no power is extracted. A similar result follows if trying to obtain the full open-circuit voltage, in which case there is no current. At maximum power conditions, both quantities are half of these values, and the load and source resistances each consume the same power.

③ Source transformation. (all ⊕ are IR; all ⊕ with an arrow are I, all resistors R)



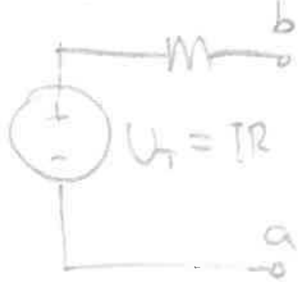






Equivalent

$R_T = R$



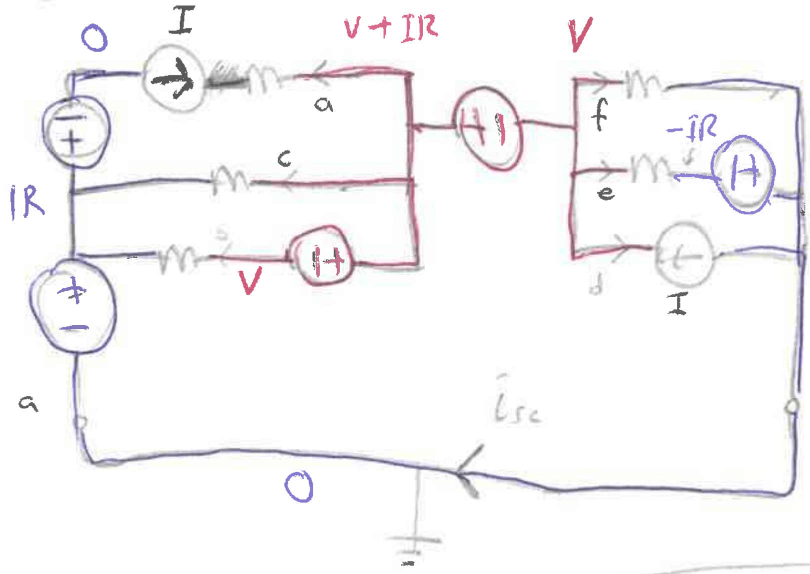
open circuit: $U_{ba} = IR$ (KVL $i=0$)

resistance (sources set to zero): R

Maximum power from this is $\frac{1}{4} U_{oc} i_{sc} = \frac{1}{4} \cdot IR \cdot \frac{IR}{R} = \frac{I^2 R}{4}$

open circuit · short circuit

③ alternative method



With shorted terminals, there are just two supernodes (and one other little node hiding at top left)

One nodal approach:

find short circuit current:

- Short the terminals
- Set something as zero
- identify supernodes
- Solve for potential(s) that let i_{sc} be found

KCL non-ground supernode:

$$0 = \underset{(a)}{-I} + \underset{(b)}{\frac{V-IR}{R}} + \underset{(c)}{\frac{V+IR-IR}{R}} \underset{(d)}{- I} + \underset{(e)}{\frac{V-(-IR)}{R}} + \underset{(f)}{\frac{V}{R}}$$

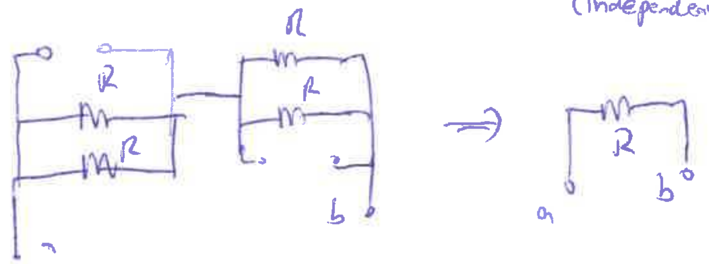
$$\Rightarrow V = \frac{IR}{2}$$

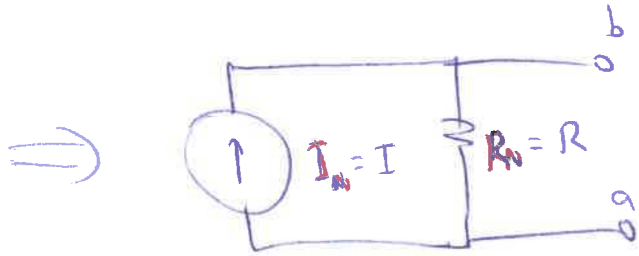
Now find i_{sc} by putting $V = \frac{IR}{2}$ into (d) + (e) + (f):

$$i_{sc} = \frac{\frac{IR}{2} - 0}{R} + \frac{\frac{IR}{2} - (-IR)}{R} - I$$

$$\boxed{i_{sc} = I}$$

Find equivalent resistance: zero all sources (independent) and simplify the resistance





source transformation