Permitted material: Beyond writing-equipment, up to three pieces of paper up to A4 size can be brought, with free choice of content: handwritten, printed; small, large; text, diagram, image; one or both sides, etc. These papers do not need to be handed in with the exam.

Unless it is stated otherwise, the final answer to a question should be expressed in terms of the known quantities given in the question, and any clear simplifications should be done. Component values such as R for a resistor, U for an independent voltage source, or K for a dependent source, are assumed to be known quantities. Marked currents or voltages such as i_x are assumed to be definitions, not known quantities.

Clearly drawn and labelled diagrams are a good way to help yourself avoid mistakes, and to make clear to others what you are doing. By showing clearly your intermediate steps in a solution, you improve your chance of getting points even if the final result is wrong. You may write in Swedish or English; but we suggest that writing in either is seldom necessary if you make good use of diagrams and equations!

Determination of exam grade. Denote as A, B and C the available points from sections A, B and C of this exam: A=12, B=10, C=18. Denote as a, b and c the points actually obtained in the respective sections, and as a_k and b_k the points från KS1 and KS2, and as h the homework 'bonus'. The requirement for passing the exam (E or higher) is:

$$\frac{\max(a, a_\mathrm{k})}{A} \geq 40\% \quad \& \quad \frac{\max(b, b_\mathrm{k})}{B} \geq 40\% \quad \& \quad \frac{c}{C} \geq 40\% \quad \& \quad \frac{\max(a, a_\mathrm{k}) + \max(b, b_\mathrm{k}) + c + h}{A+B+C} \geq 50\%$$

The grade is then determined by the total including bonus, i.e. the last of the terms above: boundaries (%) are 50 (E), 60 (D), 70 (C), 80 (B), 90 (A). If the exam misses a pass by a small margin on just one criterion, a grade of Fx may be registered, with the possibility of completing to E by an extra task arranged later.

For this VT19 round, students who have their final project-task approved will get full points on Q9 in this exam.

Nathaniel Taylor (08 790 6222)

Section A. Direct Current

---

1) [4p]

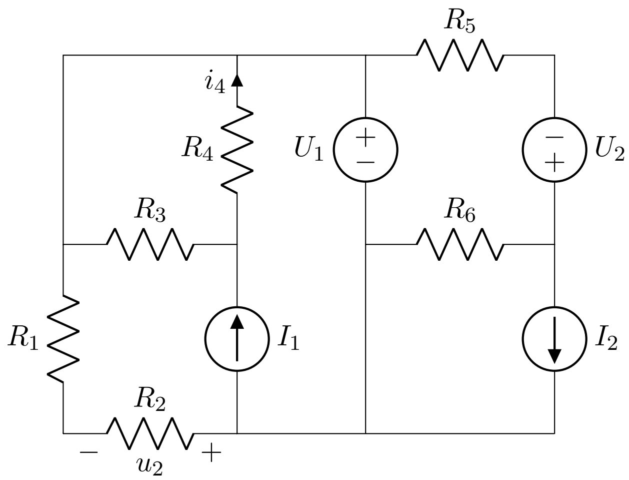
Find:

a) [1p] the current i_4

b) [1p] the voltage u_2

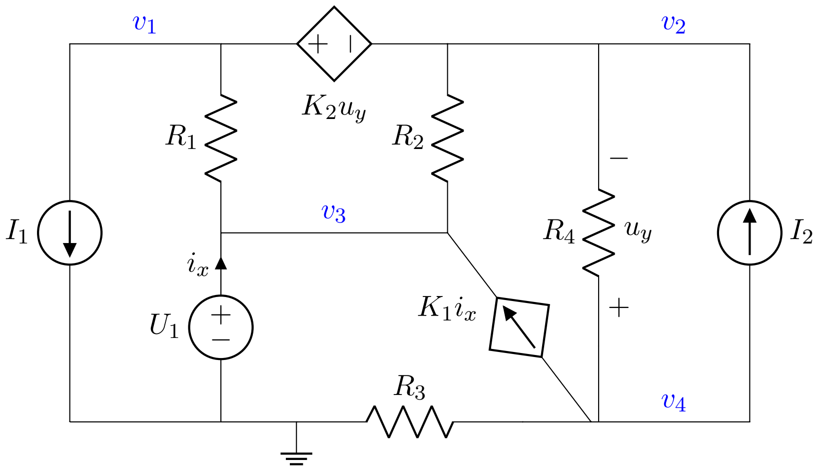
c) [1p] the power delivered from source I_1

d) [1p] the power delivered from source U_1



2) [4p]

Write equations that could be solved without further information to find the potential v_1 in terms of the component values.



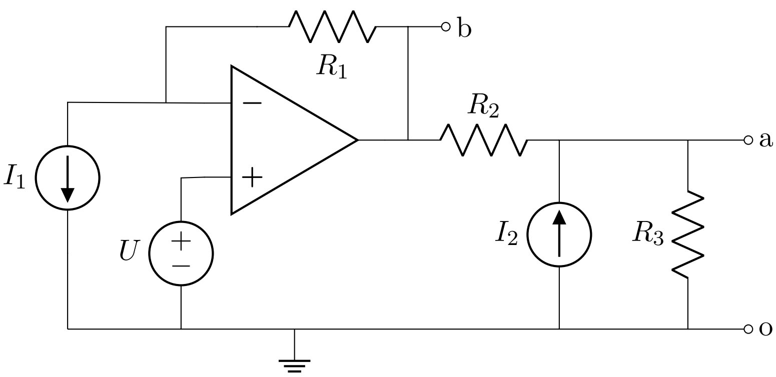
3) [4p]

What is the maximum power that can be obtained from

a) [3p] terminals a-o

b) [1p] terminals a-b

of this circuit?



Section B. Transient Calculations

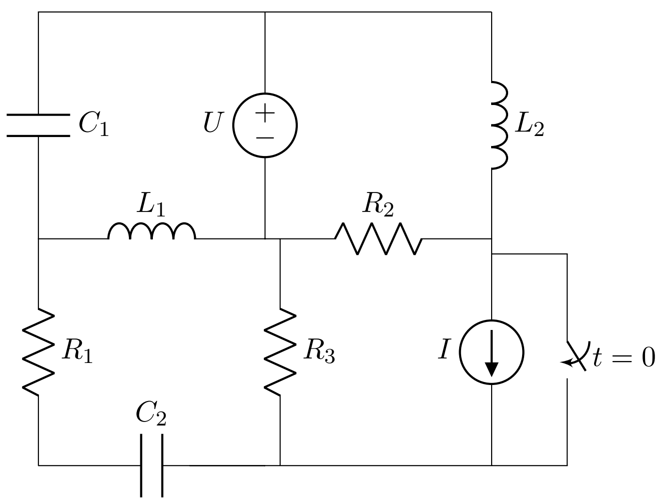
4) [5p] Find:

a) [1p] Power absorbed by R_2 at $t = 0^-$

b) [1p] Energy stored in L_2 at $t = 0^+$

c) [2p] Power supplied by C_1 at $t = 0^+$

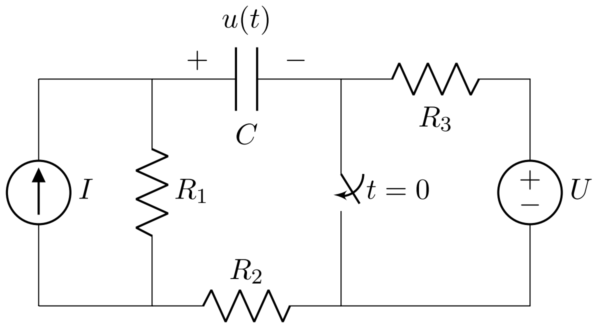
d) [1p] Energy stored in C_2 as $t \to \infty$



5) [5p]

a) [4p] Find the voltage $u(t)$, for $t > 0$.

b) [1p] Find the power absorbed by R_2 for $t > 0$.



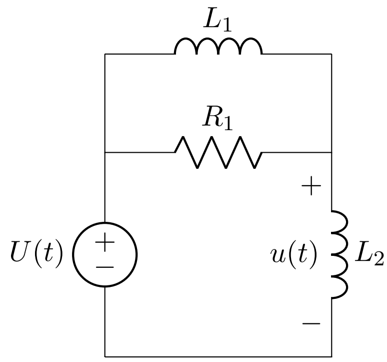
Both of the above are expected to be functions of time.

Section C. Alternating Current

6) [4p]

The source's voltage is $U(t) = \hat{U}\sin(\omega t)$.

Determine $u(t)$.



7) [4p]

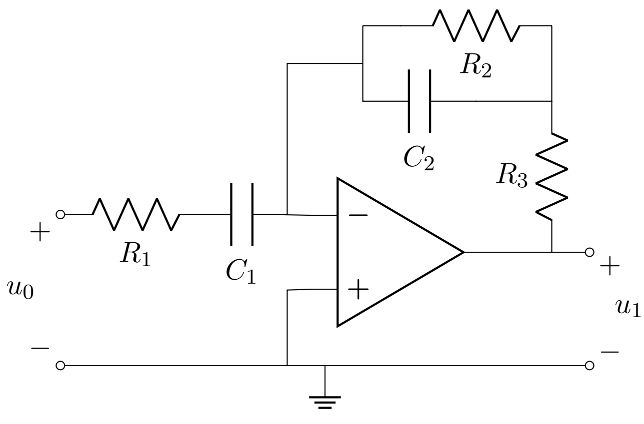
a) [2p] Determine this circuit's network function,

$$H(\omega) = \frac{u_1(\omega)}{u_0(\omega)}.$$

b) [1p] Show that the solution of 'a' can be written in the form

$$H(\omega) = \frac{-\mathrm{j}\omega/\omega_0\ (1+\mathrm{j}\omega/\omega_3)}{(1+\mathrm{j}\omega/\omega_1)\ (1+\mathrm{j}\omega/\omega_2)}.$$

It is sufficient to show how to express the parameters $\omega_{0,1,2,3}$ in terms of the circuit component values.

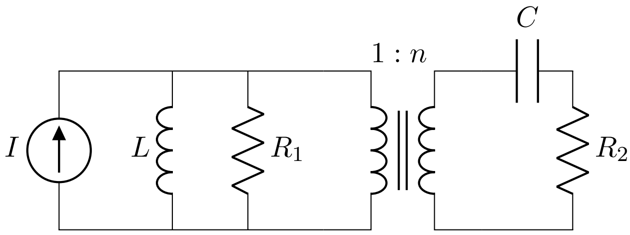


c) [1p] Sketch a Bode amplitude plot of the function $H(\omega)$ shown in 'b'.
Assume $\omega_0 \ll \omega_1 \ll \omega_2 \ll \omega_3$.
Mark the gradients (other than zero) and the frequencies ω_0 etc.

8) [4p]

The source has angular frequency ω.

Component values n and C can be chosen, but other component values are fixed.



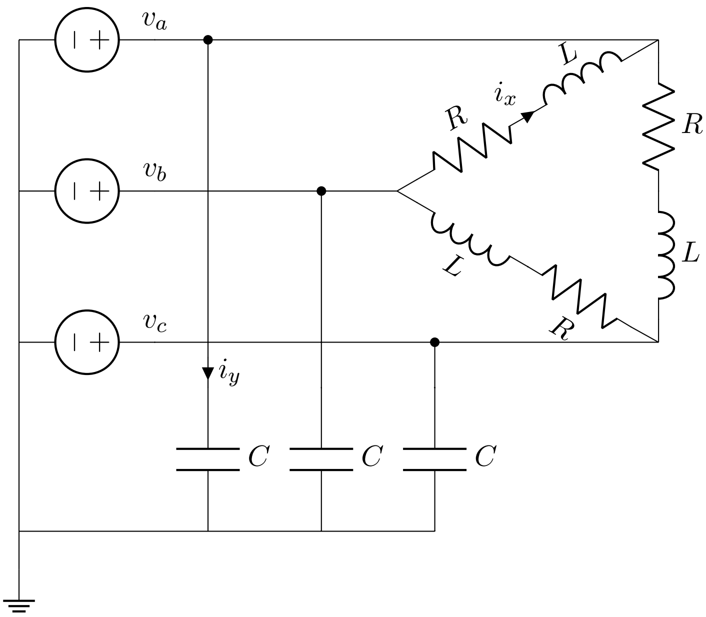
a) [3p] Determine the values of n and C that will maximise the power delivered to resistor R_2.

b) [1p] What is the value of this maximum power to R_2?

---

9) [6p]

At the left of this circuit is a balanced three-phase source, of *line*-voltage U, angular frequency ω, and phase-rotation a,b,c. The phase of v_a is taken as the reference: $\angle v_a = 0$.



a) [2p] What apparent power is supplied by the source?

b) [1p] What is i_x as a phasor (magnitude and angle)?

c) [1p] What value of capacitance C is needed in order for the source to supply purely active power?

d) [2p] The first phase ('a') of the source explodes. In its new state, the circuit can be modelled by replacing the uppermost voltage-source in the diagram by an open-circuit.
What now is i_y (magnitude and angle)?

The End. *Please don't waste remaining time ... check your solutions!*

Översättningar:

Hjälpmedel: Upp till tre A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text, diagram, bild; stor eller liten textstorlek, o.s.v. Dessa måste inte lämnas in med skrivningarna.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, K för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas.

Var tydlig med diagram och definitioner av variabler. Du får skriva på svenska eller engelska, men vi rekommenderar att diagram och ekvationer används i stället i de flesta fall.

1. [4p] Bestäm följande:
a) [1p] strömmen i_4
b) [1p] spänningen u_2
c) [1p] effekten levererad från källan I_1
d) [1p] effekten levererad från källan U_1

2. [4p] Skriv ekvationer som skulle kunna lösas, utan vidare information, för att bestämma potentialen v_1 som funktion av kretsens komponentvärden.

3. [4p] Vilken maximaleffekt kan levereras
från
a) [3p] polerna a-o
b) [1p] polerna b-a
av kretsen?

4. [5p] Bestäm:
a) [1p] Effekten absorberad av R_2 vid $t = 0^-$.
b) [1p] Energin lagrad i L_2 vid $t = 0^+$.
c) [2p] Effekten försörjd av C_1 vid $t = 0^+$.
d) [1p] Energin lagrad i C_2 vid $t \to \infty$.

5. [5p]
a) [4p] Bestäm spänningen $u(t)$ vid $t > 0$.
b) [1p] Bestäm effekten absorberad av R_2 vid $t > 0$.

6. [4p] Källans spänning är $U(t) = \hat{U} \sin(\omega t)$. Bestäm $u(t)$.

7. [4p]
a) [2p] Härled kretsens nätverksfunktion, $H(\omega) = u_1(\omega)/u_0(\omega)$.
b) [1p] Visa att funktionen från deltal 'a' kan skrivas $H(\omega) = \frac{-j\omega/\omega_0\,(1+j\omega/\omega_3)}{(1+j\omega/\omega_1)\,(1+j\omega/\omega_2)}$.
c) [1p] Skissa ett Bodeamplituddiagram av $H(\omega)$ från deltal 'b'. Antag $\omega_0 \ll \omega_1 \ll \omega_2 \ll \omega_3$. Markera viktiga punkter och lutningar.

8. [4p] Källan har vinkelfrekvens ω. Komponentvärden n och C kan väljas men andra komponentvärden är fasta.
a) [3p] Bestäm värden n och C som ger maximaleffekt till R_2.
b) [1p] Hur mycket effekt blir den till R_2 vid situationen från deltal 'a'?

9. [6p] Till vänster i kretsen är en balanserad trefas källa, med huvudspänning U, vinkelfrekvens ω, och fasföljd a,b,c. Fasvinkeln av v_a tas som referens, d.v.s. $\angle v_a = 0$.
a) [2p] Vilken skenbareffekt levererar källan?
b) [1p] Bestäm i_x (som fasvektor – magnitud och vinkel).
c) [1p] Vilken kapacitans C behövs för att källan matar rent aktiveffekt.
d) [2p] Första fasen (a) i källan exploderar. Situationen efteråt kan modelleras genom att ersätta den övre spänningskällan vid v_a med en öppenkrets. Bestäm i_y (magnitud och vinkel).

Solutions (EI1120 TEN1 VT19, 2019-03-15)

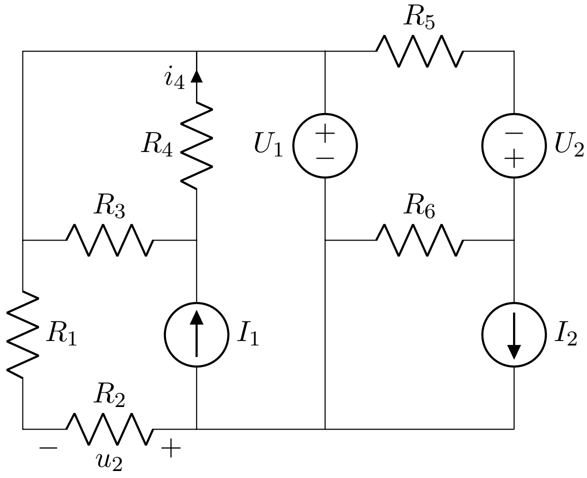
Q1.

a) $i_4 = \dfrac{R_3}{R_3 + R_4} I_1$

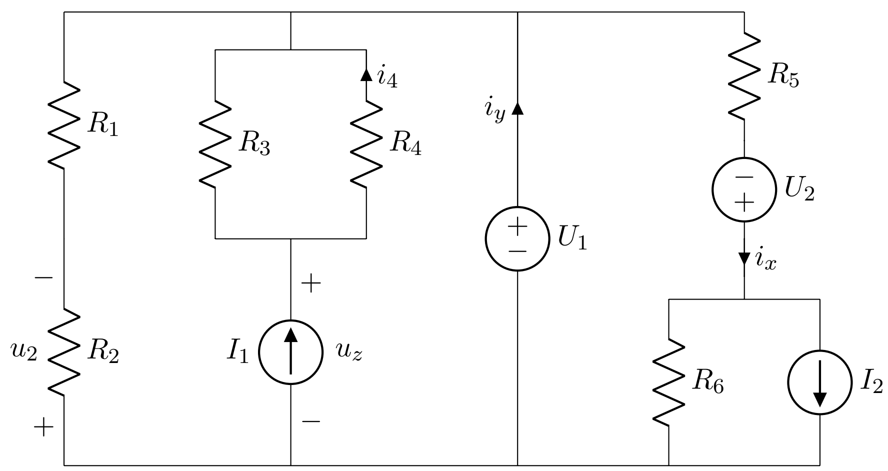
b) $u_2 = \dfrac{-R_2}{R_1 + R_2} U_1$

c) $P_{\text{I1}} = \left(U_1 + \dfrac{R_3 R_4 I_1}{R_3 + R_4} \right) I_1$

d) $P_{\text{U1}} = \left(\dfrac{U_1 + U_2 + I_2 R_6}{R_5 + R_6} + \dfrac{U_1}{R_1 + R_2} - I_1 \right) U_1$



The following is one way in which the original circuit can be re-drawn to be a bit clearer.
Some further quantities have been marked here for use during the solutions.



The current i_4 is found by current-division of the fixed source-current I_1 between the parallel resistors R_3 and R_4.

The voltage u_2 is found by voltage division between R_1 and R_2. The voltage across this series pair is U_1, which can be seen from KVL. A negative sign is needed due to the direction in which u_2 is marked relative to the voltage U_1 across the pair of resistors.

The power delivered by source I_1 is found by finding the voltage u_z across this source, and multiplying it by the source's value (current). In order for this product to give the power *from* the source, u_z must be defined in the direction shown in the diagram above; otherwise a negative sign is needed.

By KVL around the loop of { I_1, $R_3||R_4$, U_1 }, this voltage is

$$u_z = U_1 + \frac{R_3 R_4 I}{R_3 + R_4}.$$

The power delivered by source U_1 is $P_{U1} = U_1\, i_y$.

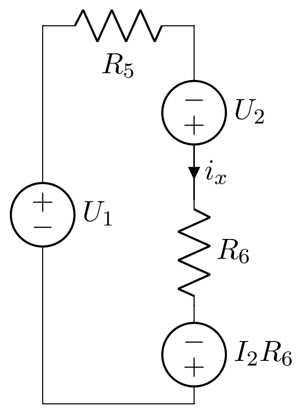
By KCL above source U_1,

$$i_y = \frac{U_1}{R_1 + R_2} - I_1 + i_x.$$

It is just the i_x term that is a bit awkward to find.

The voltage across the rightmost branch of the circuit (R_5, U_2, R_6, I_2) is determined by source U_1, and is not affected by the branches further to the left.

The circuit on the right shows the part of the original circuit relevant to finding i_x, *after* a Norton-Thevenin source-transformation on the pair { R_6, I_2 }.



From this circuit, by KVL and Ohm's law,

$$i_x = \frac{U_1 + U_2 + I_2R_6}{R_5 + R_6}.$$

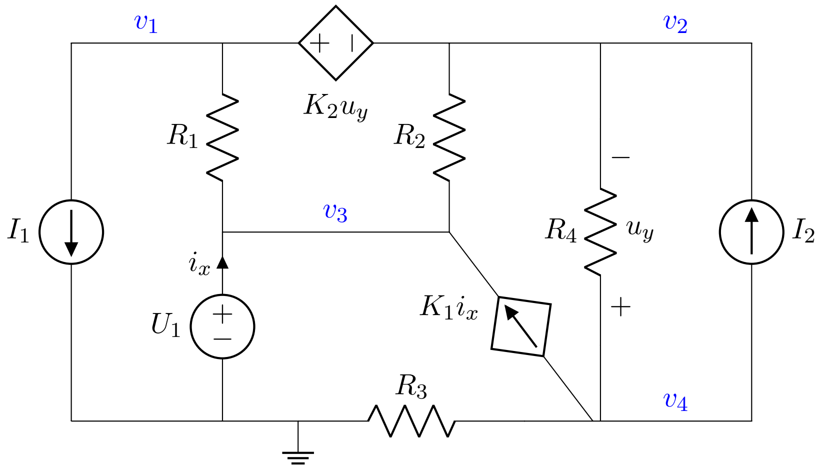
Hence,

$$P_{U1} = \left(\frac{U_1 + U_2 + I_2R_6}{R_5 + R_6} + \frac{U_1}{R_1 + R_2} - I_1\right) U_1.$$

Q2.

There isn't any particularly nice step-by-step method apparent for this circuit, so it's fortunate we only have to write suitable equations, rather than having to solve all the way for v_1.

Two possible methods are shown below: the extended nodal analysis, and the method based on supernodes and avoiding defining extra variables. The former is almost certainly easier to write, although the latter is probably easier to solve.



Extended nodal analysis.
Simple rules to follow for writing the equations, but not so nice to solve!

Start with KCL at all nodes except the reference.
This circuit has two voltage sources: one independent and one dependent. Their currents are not initially known, so we define them: call them i_α in U_1, and i_β in Ku_y, into the +-terminals. There is already a current i_x defined in the source U, but this time we'll choose to define our own current i_α separately.

$$\text{KCL(1):} \quad 0 = I_1 + \frac{v_1 - v_3}{R_1} + i_\beta \tag{1}$$

$$\text{KCL(2):} \quad 0 = -i_\beta + \frac{v_2 - v_3}{R_2} + \frac{v_2 - v_4}{R_4} - I_2 \tag{2}$$

$$\text{KCL(3):} \quad 0 = i_\alpha + \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_2}{R_2} - K_1 i_x \tag{3}$$

$$\text{KCL(4):} \quad 0 = \frac{v_4}{R_3} + K_1 i_x + \frac{v_4 - v_2}{R_4} + I_2 \tag{4}$$

The above are 4 equations, in 7 unknowns. The 4 unknown node-potentials and 4 KCL equations would give a well defined solution. But the 2 voltage sources have given further unknowns, of their currents; this hints that we should look to the voltage sources to provide corresponding further equations. The sources set the following relation between pairs of node-potentials:

$$v_3 = U_1 \tag{5}$$
$$v_1 - v_2 = K_2\, u_y \tag{6}$$

Now one further unknown, u_y, has been introduced. There are still two more unknowns than equations: these are due to the marked quantities i_x and u_y, which are the controlling variables of the two dependent sources. They have to be defined as equations, in order to convey the same information as the diagram tells us about them; otherwise the equations don't provide enough information to solve the shown circuit.

$$i_x = -i_\alpha \tag{7}$$
$$u_y = v_4 - v_2 \tag{8}$$

The above equations (1)–(8) are a sufficient solution.

Nodal analysis: simplify on the way, e.g. supernode

Another approach is to try to reduce the number of equations from the start, instead of ending up as in the above example, with lots of simpler equations to solve.

If we follow this principle, and use the idea of supernodes, then we end up with just two equations to solve; after solving them, other potentials could be found by simple relations given by the voltage sources. First we'll do the preparation work of choosing which potentials to keep, on the way to writing the KCL equations.

The nodes 0 and v_3 are joined into a supernode by the independent voltage source; they are a 'ground supernode'. Instead of using the potential v_3 in equations, we therefore substitute

$$v_3 = U_1. \tag{1}$$

Nodes v_1 and v_2 are joined by the dependent voltage source, giving the relation $v_1 = v_2 + K_2\, u_y$. We usually try to avoid marked quantities such as u_y in the equations (see later), so looking at the diagram we substitute for this in terms of node potentials, $u_y = v_4 - v_2$, leading to

$$v_1 = (1 - K_2)\, v_2 + K_2\, v_4. \tag{2}$$

Only one of the supernode's potentials v_1 or v_2 needs to be kept as an unknown in the equations. It could seem good to keep v_1, since this is what we're actually asked to find. However, in this case a little experimenting suggests that it's easier to write the equations if it's v_2 that's defined. So we'll keep v_2, and substitute from (2) wherever v_1 tries to appear in a KCL or other equation.

If our aim is to write our KCLs without including further unknowns that need further equations, then we should avoid using the controlling variables i_x and u_y in the equations. From the diagram, it is easy to express the latter as

$$u_y = v_4 - v_2,$$

which would have been messier if v_1 instead of v_2 had been chosen as the potential to keep in the supernode. The current i_x is harder: it can be found from KCL in node 3,

$$i_x = -K_1\, i_x + \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_1}{R_1},$$

but this introduces a further i_x term and also a v_1 and v_3, which are not the node potentials we decided to keep, and therefore have to be substituted. Putting it together,

$$i_x = \frac{\frac{U_1 - v_2}{R_2} + \frac{U_1 - ((1-K_2)v_2 + K_2 v_4)}{R_1}}{1 + K_1} = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) U_1 - \left(\frac{1-K_2}{R_1} + \frac{1}{R_2}\right) v_2 - \frac{K_2 v_4}{R_1}}{1 + K_1}.$$

Now we can write the two necessary KCL equations: one for the supernode that contains the nodes marked v_1 and v_2, and one for the node marked v_4.

$$\begin{aligned}\text{KCL(1\&2):} \quad 0 &= I_1 + \frac{(1-K_2)\,v_2 + K_2\,v_4 - U_1}{R_1} + \frac{v_2-v_3}{R_2} + \frac{v_2-v_4}{R_4} - I_2 \qquad (3)\\ \text{KCL(4):} \quad 0 &= \frac{v_4}{R_3} + \frac{v_4-v_2}{R_4} + I_2 + K_1 \frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)U_1 - \left(\frac{1-K_2}{R_1}+\frac{1}{R_2}\right)v_2 - \frac{K_2 v_4}{R_1}}{1+K_1} \qquad (4)\end{aligned}$$

The equations (3) and (4) are part of the solution, and must be solved together. The equations (1) and (2) are the remainder of the solution, and are necessary in order to make up an equation system that can be solved for all node potentials.

As this question only required that v_1 can be solved for, one could omit (1).

The 'moral' of the above is probably that the extended method was much easier to write for this circuit. It has the further advantage that if the circuit is changed, the corresponding change can easily be made in the equations, since each equation directly describes some feature of the circuit without being obfuscated by substitutions and rearrangements.

Various checks can be done, symbolically or numerically, to compare the above solutions with each other or with another calculation. Confession: in writing the extended-method equations, I initially got the wrong sign on i_α, having looked at the arrow marked for i_x; checking *is* a worthwhile effort.

Below we use Matlab's symbolic toolbox to compare the above two sets of equations symbolically. Then we subsitute numbers into the symbolic solutions and compare the result with a calculation by the program `SPICE 2g.6` (from 15/March/1983!).

```
syms  U1 I1 I2 K1 K2 R1 R2 R3 R4
syms  v1 v2 v3 v4
syms  ix uy

% extended nodal analysis
syms ia ib
s1 = solve( ...
    { ...
     0 == I1 + (v1-v3)/R1 + ib, ...
     0 == -ib + (v2-v3)/R2 + (v2-v4)/R4 - I2, ...
     0 ==  ia + (v3-v1)/R1 + (v3-v2)/R2 - K1*ix, ...
     0 == v4/R3 + K1*ix + (v4-v2)/R4 + I2, ...
     v3 == U1, ...
     v1 - v2 == K2*uy, ...
     ix == -ia, ...
     uy == v4 - v2 ...
     }, ...
     { v1, v2, v3, v4, ix, uy, ia, ib }   );

% supernode
s2 = solve( ...
    { ...
        0 == I1 + ( (1-K2)*v2 + K2*v4 - U1 )/R1 + (v2-v3)/R2 + (v2-v4)/R4 - I2, ...
        0 == v4/R3 + (v4-v2)/R4 + I2 + ...
              K1*( (1/R1 + 1/R2)*U1 - ((1-K2)/R1 + 1/R2)*v2 - K2*v4/R1 )/(1+K1), ...
        v3 == U1, ...
        v1 == (1-K2)*v2 + K2*v4 ...
     }, ...
     { v1, v2, v3, v4 }   );

%% symbolic check
simplify( s1.v1 - s2.v1 )
simplify( s1.v2 - s2.v2 )
simplify( s1.v3 - s2.v3 )
simplify( s1.v4 - s2.v4 )
  % --> all zero :  good
```

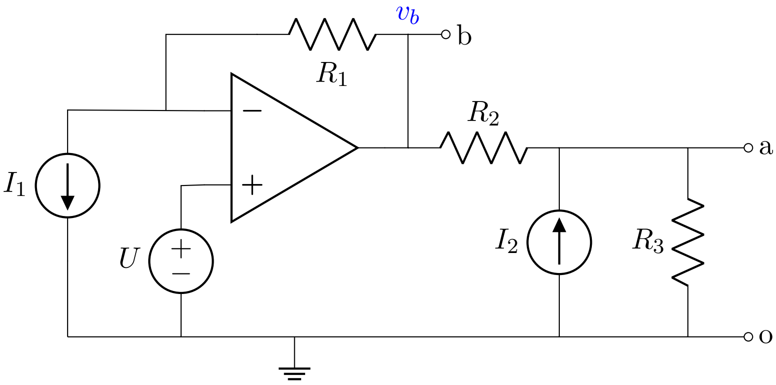
```
%% numeric check
U1=27; I1=1; I2=5; K1=0.2; K2=0.11; R1=20; R2=13; R3=3; R4=40;
for fld={'v1','v2','v3','v4'},
   fprintf(' %s = %7.4f \n', fld{1}, double(subs(s1.(fld{1}))) );
end
%    v1 = 43.0120
%    v2 = 49.4805
%    v3 = 27.0000
%    v4 = -9.3247


%  Input "netlist" file for SPICE
%
EI1120_VT19_TEN1_Q2
V1      3  0   DC   27.0
I1      1  0   DC    1.0
I2      4  2   DC    5.0
F1      4  3   V1   -0.2
E1      1  2  4 2    0.11
R1      1  3        20.0
R2      2  3        13.0
R3      4  0         3.0
R4      4  2        40.0
.OP
.PRINT DC V(0) V(1) V(2) V(3) V(4)
.END

% output:
%  node   voltage     node   voltage     node   voltage     node   voltage
%  (  1)   43.0120    (  2)   49.4805    (  3)   27.0000    (  4)   -9.3247
```

Q3.

The opamp's output behaves as a voltage source. Its potential, v_b, will be whatever value is necessary in order for the inverting input to have the same potential as the non-inverting input, which is U.

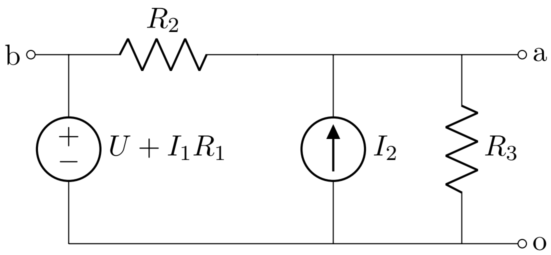
KCL at the inverting input gives $I_1 = (v_b - U)/R_1$, from which

$$v_b = U + I_1 R_1.$$



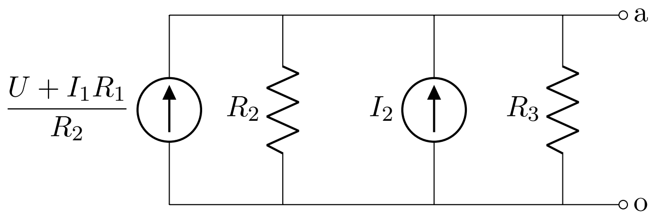
Seen by the components to the right of the opamp output, the opamp output therefore behaves as a fixed voltage source v_b, with its other side connected to the reference node. For solving the circuit at the right, we can represent the opamp and its feedback and inputs as a voltage source $U + I_1R_1$, leading to the following circuit.

(Note that replacing the opamp with a fixed voltage source is valid and useful because we're interested in what happens on the right of the opamp, and we are not considering changing anything in its feedback and inputs – if we considered connecting other things to 'extract power' from the parts around the left, that might change the circuit so the opamp would have a different voltage.)



a)

Between terminals a-o, we can find a Norton equivalent by doing source transformation on the voltage source and adjacent resistor from the above diagram, leading to the circuit on the right.



Between a-o this simplifies to a Norton source of

$$I_{\mathrm{N}} = \frac{U}{R_2} + \frac{R_1}{R_2}I_1 + I_2 \qquad \text{and} \qquad R_{\mathrm{N}} = \frac{R_2R_3}{R_2+R_3}.$$

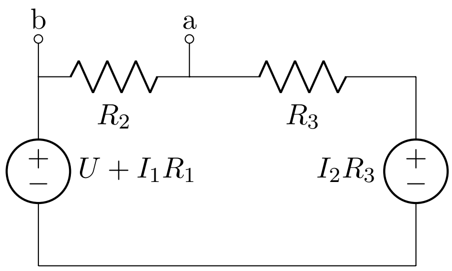
The maximum power that such a source can supply is

$$P_{\mathrm{max:ao}} = \frac{1}{4}I_{\mathrm{N}}^2 R_{\mathrm{N}} = \frac{1}{4}\left(\frac{U}{R_2} + \frac{R_1}{R_2}I_1 + I_2\right)^2 \frac{R_2R_3}{R_2+R_3} = (U + I_1R_1 + I_2R_2)^2 \frac{R_3}{4R_2(R_2+R_3)}.$$

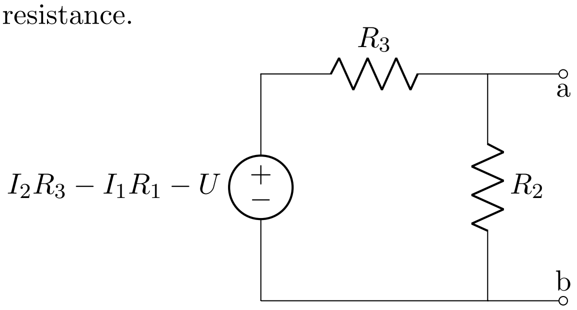
b)

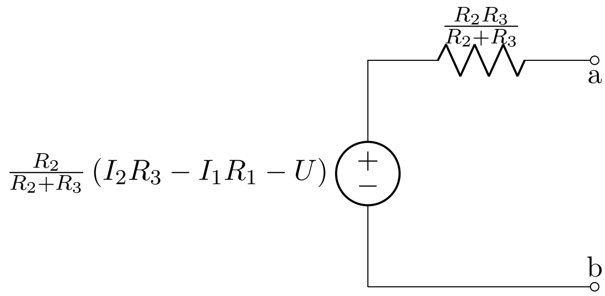
The terminal 'b' is at a node that disappeared when doing the source transformation in question 'a' above. That method is therefore not directly useful now.

Instead, we can find a Thevenin equivalent by source transformation of current source I_2 and its parallel resistor R_3, to give the circuit shown on the right.



This simplifies to the circuit at the left below, by combining the two series sources into an equivalent one and re-drawing. That circuit in turn simplifies to a Thevenin source, by voltage division and parallel resistance.





The Thevenin equivalent between a-b is therefore

$$U_{\mathrm{T}} = \frac{R_2}{R_2+R_3}(I_2R_3 - I_1R_1 - U) \qquad \text{and} \qquad R_{\mathrm{T}} = \frac{R_2R_3}{R_2+R_3}.$$

The maximum power that such a source can supply is

$$P_{\mathrm{max:ab}} = \frac{U_{\mathrm{T}}^2}{4R_{\mathrm{T}}} = \frac{\left(\frac{R_2}{R_2+R_3}\right)^2 (I_2R_3 - I_1R_1 - U)^2}{4\frac{R_2R_3}{R_2+R_3}} = \frac{R_2\,(I_2R_3 - I_1R_1 - U)^2}{4R_3\,(R_2+R_3)}$$

Both 'a' and 'b' could have been done instead by other methods, such as nodal analysis, superposition, etc. Since I seem to be in a mood for writing lots of diagrams in this year's solutions, I'm trying more of the 'intuitive' step-by-step solutions.

Q4.

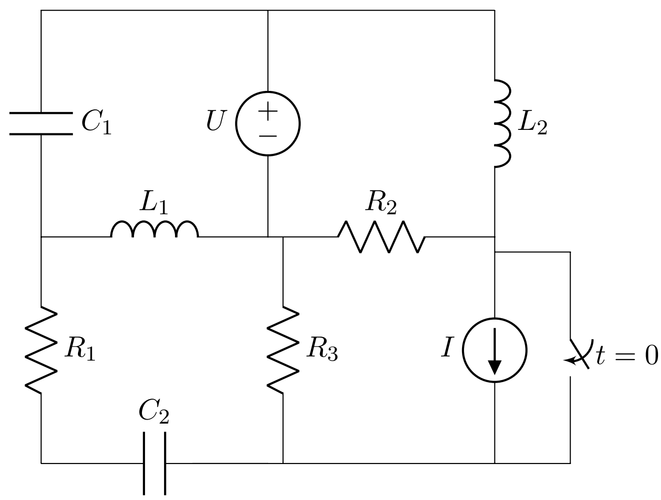
Task — find:

a) $P_{\mathrm{R2}}(0^-) = U^2/R_2$

b) $W_{\mathrm{L2}}(0^+) = \frac{1}{2}L_2\left(\frac{U}{R_2} + I\right)^2$

c) $P_{\mathrm{C1}}(0^+) = \frac{(U - IR_3)UR_3}{R_1R_2 + R_2R_3 + R_3R_1}$.

d) $W_{\mathrm{C2}}(\infty) = \frac{1}{2}C_2U^2$



At time $t = 0^-$, the switch is still open-circuit.

No change has happened in the circuit yet, so equilibrium is assumed. This means that inductors have no voltage, capacitors have no current

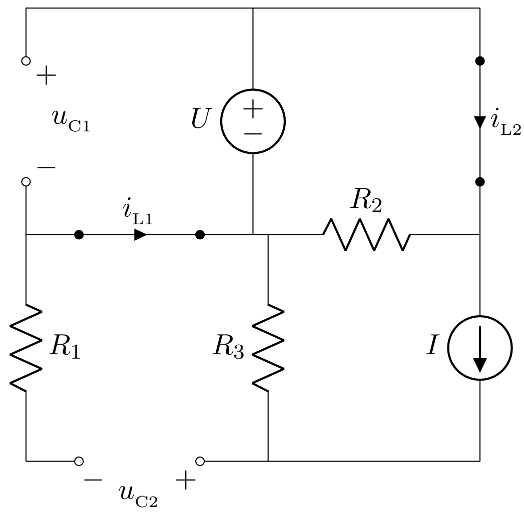
The circuit below shows the situation for $t = 0^-$, with the switch open, inductors short-circuited and capacitors open-circuited. All the inductor currents and capacitor voltages are marked; these may be useful when we come to $t = 0^+$, so we will find all of them now.

$i_{L1} = 0$
KCL, with zero current at the open-circuited capacitors.

$i_{L2} = \frac{U}{R_2} + I$
KVL to find voltage across R_2, then Ohm's law for its current, then KCL above source I.

$u_{C1} = U$
KVL around the top left loop.

$u_{C2} = IR_3$
KCL below R_3, Ohm's law in R_3, and KVL around the bottom left loop.



Question 'a' requires the power absorbed by resistor R_2 in this equilibrium state. By KVL around the top right loop, the voltage across this is U, so the power is U^2/R_2.

At time $t = 0^+$, the switch is closed, and inductors and capacitors cannot be assumed to be in equilibrium any more, since a change has happened.

The switch short-circuits the current source. Seen from all the rest of the circuit, the current source is 'invisible' (irrelevant): whatever current it produces just circulates in the one node that it's connected to. So we're probably best to remove it from the diagram for clarity, since none of the questions wants to know something about it such as what power it produces or what voltage it has . . . both of which are zero.

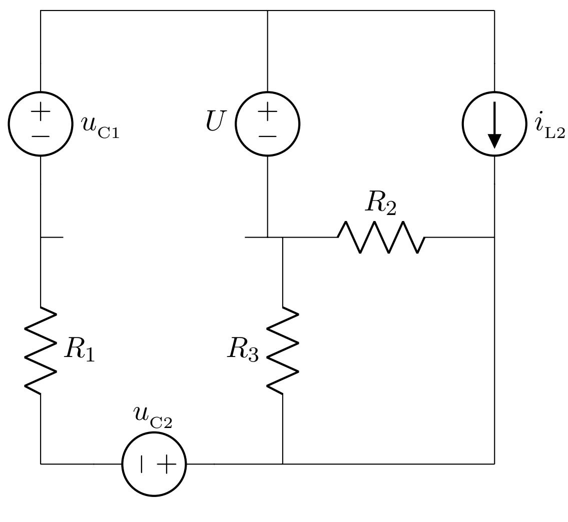
The capacitors and inductors have known continuous variables at $t = 0^+$, since these must be the same as at $t = 0^-$. They are therefore modelled here as fixed sources, whose values were found above for $t = 0^-$. To begin with we'll leave them as neat symbols such as u_{C1}.

As inductor L_1 was found to have zero current it is more simply modelled as an open-circuit. Simplicity is key to improving our chance of seeing neat solutions.

The circuit below is the resulting view of the situation at $t = 0^+$.

Question 'b' wants the energy stored in the inductor L_2. Energy depends on the continuous variable, which is the same at $t = 0^+$ as at $t = 0^-$, so this question can be answered entirely from the earlier circuit.
The energy is $\frac{1}{2}L_2\, i_{L2}^2$, which is $\frac{1}{2}L_2\left(\frac{U}{R_2} + I\right)^2$.

Question 'c' wants the power delivered by the capacitor C_1. This is more difficult. The current upwards through the capacitor needs to be found, and multiplied with the voltage u_{C1}. Any of superposition, source transformation or nodal analysis could be used to find the current.

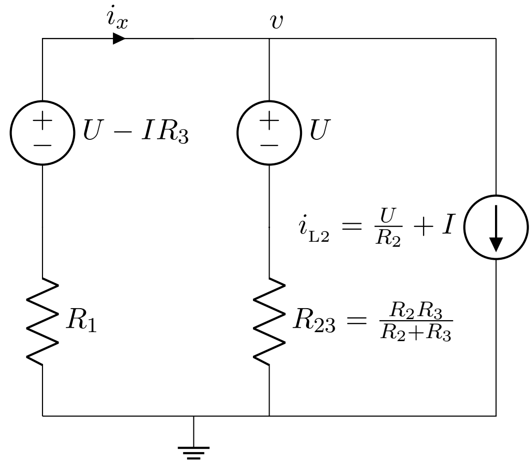


To apply nodal analysis in a simple way with one KCL, we can regard the circuit as three parallel branches, as shown in a simpler form below.

The two series-connected voltage sources have been combined to a single one of $u_{\rm C1} - u_{\rm C2}$.

Then the values calculated at $t = 0^-$ have been substituted to give $U - IR_3$ for that source, and $\frac{U}{R_2} + I$ for the source on the right representing L_2.

The resistors R_2 and R_3 were in parallel, so have been combined also.



Using the marked potential v, KCL gives

$$\frac{v - U + IR_3}{R_1} + \frac{v - U}{R_2 R_3}(R_2 + R_3) + \frac{U}{R_2} + I = 0$$

After some effort, a solution for v is found,

$$v = \frac{R_2 (R_1 + R_3)(U - IR_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3},$$

from which in turn the current i_x is found,

$$i_x = \frac{U - IR_3 - v}{R_1} = \frac{(U - IR_3) R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

The above was quite a lot of effort with unwieldy expressions for component values. It added to the effort that we did it by the canonical node-potential method, then found i_x from v.

One rather neat 'more intuitive' alternative method is the following:

> We want to find i_x in the above circuit. Imagine breaking the circuit (introducing an open-circuit) at the point where i_x is marked, and finding the Thevenin equivalent between the two sides of the break.
>
> The Thevenin resistance can be found by setting sources to zero and simplifying the remaining resistors, which gives $R_{\rm T} = R_1 + R_{23}$.
>
> The Thevenin voltage of the left relative to the right side of the break is $U - IR_3 + i_{\rm L2} R_{23} - U$, which is seen from KVL in the left loop, bearing in mind that with the break in the circuit all of the current from the current source must pass up through R_{23}.
>
> Putting in the given quantities instead of our defined names,
>
> $$\begin{aligned} U_{\rm T} &= U - IR_3 + i_{\rm L2} R_{23} - U = -IR_3 + \left(\frac{U}{R_2} + I\right)\frac{R_2 R_3}{R_2 + R_3} \\ &= -IR_3 + (U + IR_2)\frac{R_3}{R_2 + R_3} = \frac{(U - IR_3) R_3}{R_2 + R_3} \end{aligned}$$
>
> What we *want* is the short-circuit current of this Thevenin source:
>
> $$i_x = \frac{U_{\rm T}}{R_{\rm T}} = \frac{\frac{(U - IR_3) R_3}{R_2 + R_3}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{(U - IR_3) R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}.$$

That felt somehow a bit more satisfying than the nodal way, but neither was trivial!

Now we find the power from the capacitor by multiplying this current by the capacitor's voltage,

$$P_{\rm C1}(0^+) = u_{\rm C1} i_x = U i_x = \frac{(U - IR_3) U R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}.$$

At time $t \to \infty$, equilibrium can again be assumed. The difference from $t = 0^-$ is that the switch is now closed, and all memory of the short-circuited current source I will now have disappeared from the circuit.

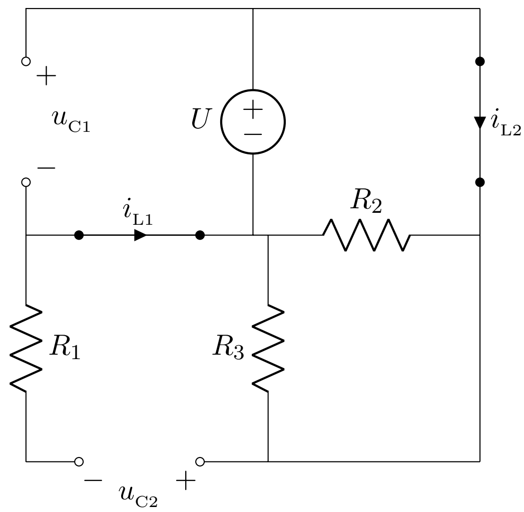
Question 'd' wants the energy stored in capacitor C_2.

This is $\frac{1}{2}C_2 u_{C2}^2$.

The voltage u_{C2} can be found by taking a KVL around $\{ C_2, R_1, L_1, U, L_2 \}$, in which only C_2 and U have non-zero voltages.

Thus the energy is $\frac{1}{2}C_2 U^2$.

A nice feature of squaring the voltage is that we didn't have to care about its direction.

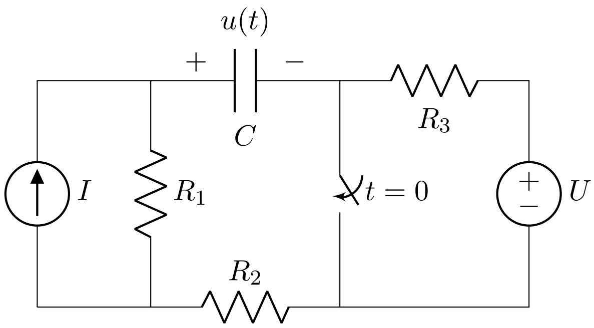


Q5.

Solutions of the circuit at the right, for $t > 0$:

a) $u(t) = IR_1 - U\,e^{-t/(R_1+R_2)C}$

b) $P_{R2}(t) = \dfrac{U^2 R_2}{(R_1+R_2)^2}\,e^{-t/(R_1+R_2)C}$

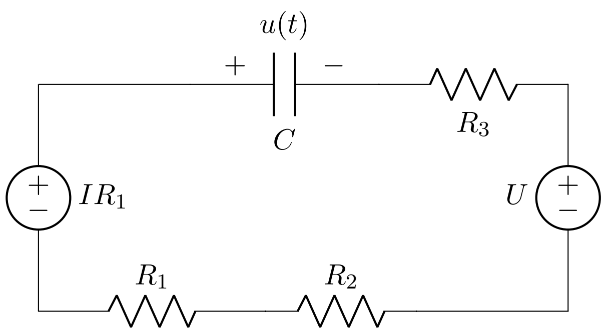


Initial conditions

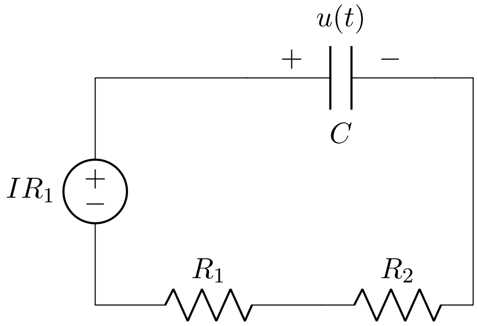
The diagram to the right shows the circuit with the switch open.

At $t = 0^-$ an equilibrium can be assumed, in which the capacitor has no current, and there is therefore no voltage drop across the resistors in the loop.

From KVL, $u(0^-) = IR_1 - U$.



Circuit at $t > 0$



When the switch closes, the circuit becomes simpler.

The capacitor's voltage is a continuous variable, so it is initially unchanged: $u(0^+) = u(0^-) = IR_1 - U$.

The rest of the circuit that the capacitor is connected to has a Thevenin equivalent of $U_{\mathrm{T}} = IR_1$ and $R_{\mathrm{T}} = R_1{+}R_2$.

As $t \to \infty$ the circuit will reach a new equilibrium, in which the capacitor's voltage will equal the Thevenin voltage.

We now know the initial value and final value of $u(t)$ for $t \geq 0$. The time-constant is CR_{T}.

Putting these into the usual exponential decay expression for first-order circuits,

$$u(t) \;=\; u(\infty) + \big(u(0^+) - u(\infty)\big)\,\mathrm{e}^{-t/\tau} \;=\; IR_1 + (IR_1{-}U - IR_1)\,\mathrm{e}^{-t\big/R_{\mathrm{T}}C}$$

$$u(t) \;=\; IR_1 - U\mathrm{e}^{-t\big/(R_1+R_2)C}.$$

The power in R_2 is i^2R_2 where i is the current through R_2. Looking at the circuit, the current is the same through R_2 and the capacitor as they are in series. The direction doesn't matter, as the current is squared to find the power.

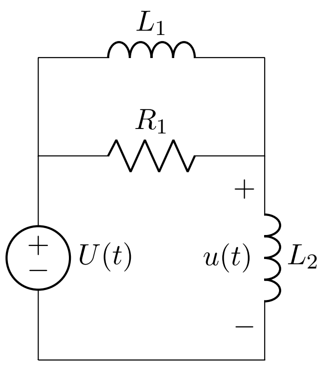
We could find $i(t)$ from $u(t)$ using KVL and Ohm's law, or by using the equation of a capacitor. Let's try the latter:

$$i \;=\; C\frac{\mathrm{d}}{\mathrm{d}t}\left(IR_1 - U\mathrm{e}^{-t\big/(R_1+R_2)C}\right) \;=\; -\frac{-UC}{(R_1+R_2)\,C}\mathrm{e}^{-t\big/(R_1+R_2)C} \;=\; \frac{U}{R_1+R_2}\,\mathrm{e}^{-t\big/(R_1+R_2)C}.$$

So,

$$P_{\mathrm{R2}} \;=\; i^2R_2 \;=\; \left(\frac{U}{R_1+R_2}\,\mathrm{e}^{-t\big/(R_1+R_2)C}\right)^2 R_2 \;=\; \frac{U^2R_2}{(R_1+R_2)^2}\,\mathrm{e}^{-2t\big/(R_1+R_2)C}.$$

Q6.



Task: determine $u(t)$, given $U(t) = \hat{U}\sin(\omega t)$.

First, represent the circuit suitably for AC analysis, with phasors and impedances.

Let's use "sine reference", so that the time-domain quantity $\sin\omega t$ is represented by a phasor with zero angle. And let's use the peak of the sinusoid as the magnitude of the corresponding phasor.

With these choices, the source becomes a phasor of $U(\omega) = \hat{U}\underline{/0}$.

The inductances are represented by their impedances, $\mathrm{j}\omega L_1$ and so forth.

Voltage division seems a sensible approach for this circuit, with the parallel combination of L_1 and R_1 as one of the two impedances in the divider:

$$u(\omega) \;=\; U(\omega)\,\frac{\mathrm{j}\omega L_2}{\mathrm{j}\omega L_2 + \frac{\mathrm{j}\omega L_1 R_1}{R_1+\mathrm{j}\omega L_1}}.$$

That's it ... except that some rearrangement is useful in order to get nicer expressions for the magnitude and angle of this result. It is necessary to have that polar description in order to write the final result for $u(t)$.

Substituting for $U(\omega)$, cancelling the $\mathrm{j}\omega$ factors, and then cancelling L_2,

$$u(\omega) \;=\; \hat{U}\,\frac{L_2}{L_2+\frac{R_1L_1}{R_1+\mathrm{j}\omega L_1}} \;=\; \hat{U}\,\frac{L_2\,(R_1+\mathrm{j}\omega L_1)}{L_2\,(R_1+\mathrm{j}\omega L_1)+R_1L_1} \;=\; \hat{U}\,\frac{R_1+\mathrm{j}\omega L_1}{\left(1+\frac{L_1}{L_2}\right)R_1+\mathrm{j}\omega L_1}.$$

Now the magnitude and angle must be found, in order to write the time-function $u(t)$. The above expression is a quotient of two rectangular complex numbers.

One approach is to find the polar form of each, and take the ratio of magnitudes and the difference in angles:

$$|u(\omega)| \;=\; \hat{U}\sqrt{\frac{R_1^2+\omega^2L_1^2}{\left(1+\frac{L_1}{L_2}\right)^2R_1^2+\omega^2L_1^2}} \qquad \underline{/u(\omega)} \;=\; \operatorname{atan}\frac{\omega L_1}{R_1}-\operatorname{atan}\frac{\omega L_1}{\left(1+\frac{L_1}{L_2}\right)R_1}$$

Another is to get a single rectangular complex number before converting, i.e. to separate real and imaginary parts. Multiply the numerator and denominator by the complex conjugate of the denominator,

$$u(\omega) \;=\; \hat{U}\,\frac{(R_1+\mathrm{j}\omega L_1)\left(\left(1+\frac{L_1}{L_2}\right)R_1-\mathrm{j}\omega L_1\right)}{\left(1+\frac{L_1}{L_2}\right)^2R_1^2+\omega^2L_1^2} \;=\; \hat{U}\,\frac{\left(1+\frac{L_1}{L_2}\right)R_1^2+\omega^2L_1^2+\mathrm{j}\omega\frac{L_1^2R_1}{L_2}}{\left(1+\frac{L_1}{L_2}\right)^2R_1^2+\omega^2L_1^2}$$

$$|u(\omega)| \;=\; \hat{U}\frac{\sqrt{\left(\left(1+\frac{L_1}{L_2}\right)R_1^2+\omega^2L_1^2\right)^2+\left(\omega\frac{L_1^2R_1}{L_2}\right)^2}}{\left(1+\frac{L_1}{L_2}\right)^2R_1^2+\omega^2L_1^2} \qquad \underline{/u(\omega)} \;=\; \operatorname{atan}\frac{\omega\frac{L_1^2R_1}{L_2}}{\left(1+\frac{L_1}{L_2}\right)R_1^2+\omega^2L_1^2}$$

The expressions from the two methods should, of course, be equivalent to each other. If one likes having just one atan function it may be better to use the second method for the angle, and the first for a neater expression for magnitude.

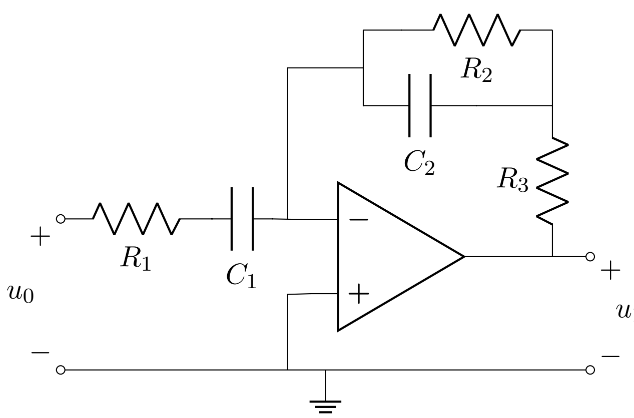
All the expressions for magnitude or angle were tediously long. It would be acceptable in the exam to write them just once, and to show how they would be used to write the time-function $u(t)$. Bearing in mind the sine-reference and peak value scale that we chose when defining the phasors, this is:

$$u(t) = |u(\omega)|\,\sin\left(\omega t+\underline{/u(\omega)}\right).$$

Note that this is exceptional — usually one should write the final answer with just the given (known) quantities. It's often possible to simplify the final expression after substituting the values of help-variables that were used during the solution. But one can't do such simplification between the magnitude and angle expressions.

Q7.

a) Determine $H(\omega)=u_1(\omega)/u_0(\omega)$.



This circuit can be seen as a standard inverting amplifier configuration, with input impedance Z_i and feedback impedance Z_f formed from the groups of resistors and capacitors,

$$Z_\mathrm{i}=R_1+\frac{1}{\mathrm{j}\omega C_1} \qquad \text{and} \qquad Z_\mathrm{f}=R_3+\frac{R_2\frac{1}{\mathrm{j}\omega C_2}}{R_2+\frac{1}{\mathrm{j}\omega C_2}}.$$

By KCL at the inverting input, or by the standard inverting-amplifier formula,

$$H(\omega) \;=\; \frac{u_1(\omega)}{u_0(\omega)} \;=\; \frac{-Z_{\rm f}}{Z_{\rm i}} \;=\; -\,\frac{R_3 + \frac{R_2 \frac{1}{{\rm j}\omega C_2}}{R_2 + \frac{1}{{\rm j}\omega C_2}}}{R_1 + \frac{1}{{\rm j}\omega C_1}}.$$

Rearranging,

$$H(\omega) \;=\; -\,\frac{\left({\rm j}\omega C_2 R_2 R_3 + R_2{+}R_3\right)\frac{1}{{\rm j}\omega C_2}}{\left(R_1 + \frac{1}{{\rm j}\omega C_1}\right)\left(R_2 + \frac{1}{{\rm j}\omega C_2}\right)} \;=\; \frac{-{\rm j}\omega C_1\,(R_2{+}R_3)\left(1 + {\rm j}\omega C_2 \frac{R_2 R_3}{R_2{+}R_3}\right)}{(1 + {\rm j}\omega C_1 R_1)\,(1 + {\rm j}\omega C_2 R_2)}.$$

b) Express $H(\omega)$ in the form $\dfrac{-{\rm j}\omega/\omega_0\;\,(1 + {\rm j}\omega/\omega_3)}{(1 + {\rm j}\omega/\omega_1)\;\,(1 + {\rm j}\omega/\omega_2)}$.

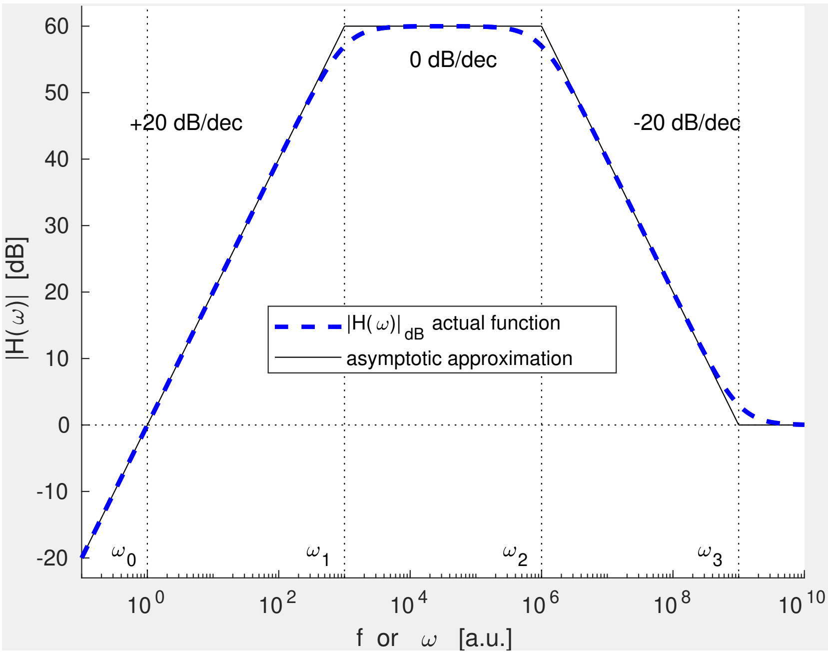
The final expression for $H(\omega)$ in part 'a' is already in a suitable form. We just need to show what values the various ω_x must have:

$$\omega_0 = \frac{1}{C_1\,(R_2 + R_3)}, \quad \omega_1 = \frac{1}{C_1 R_1}, \quad \omega_2 = \frac{1}{C_2 R_2}, \quad \omega_3 = \frac{R_2 + R_3}{C_2 R_2 R_3}.$$

The values of ω_1 and ω_2 could have been defined the opposite way round; that would show poor taste in spite of being technically correct.

c) Sketch a Bode amplitude plot of $H(\omega)$, assuming $\omega_0 \ll \omega_1 \ll \omega_2 \ll \omega_3$.

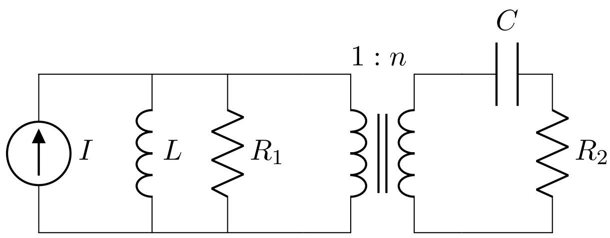
An example is shown on the right. This has a ratio 1000 between different frequencies, leading to 60 dB at the maximum. Lower ratios would give a lower maximum and a bigger deviation between the Bode approximation and the exact magnitude.



Q8.

Component values n and C can be chosen, but other component values are fixed.

This is a fairly standard maximum power question: one can identify a fixed source, and a freely variable load-impedance consisting of the transformer and the components on its right.



The slight 'twist' is that as the resistor in the load is fixed, the real part of load impedance has to be varied by the transformer ratio; then the capacitor can be chosen to give the desired imaginary part.

a) Determine the values of n and C that will maximise the power delivered to resistor R_2.

If we see the 'source' as everything to the left of the transformer, then the source impedance is

$$Z_\mathrm{s} \;=\; \frac{\mathrm{j}\omega L R_1}{R_1+\mathrm{j}\omega L} \;=\; \frac{\omega^2 L^2 R_1 + \mathrm{j}\omega L R_1^2}{R_1^2+\omega^2 L^2}.$$

The remainder of the circuit is then the load. The branch on the right of the transformer is an impedance of $R_2+\frac{1}{\mathrm{j}\omega C}$. What the source 'sees' at the transformer's left terminals is therefore this impedance scaled,

$$Z_\mathrm{l} \;=\; \frac{R_2}{n^2} - \mathrm{j}\frac{1}{n^2\omega C}$$

Now that the source and load impedances are both expressed with real and imaginary parts separated, it is easy to use the AC maximum power theorem:

$$Z_\mathrm{l} = Z_\mathrm{s}^* \qquad \Longrightarrow \qquad \frac{R_2}{n^2} - \mathrm{j}\frac{1}{n^2\omega C} \;=\; \frac{\omega^2 L^2 R_1 - \mathrm{j}\omega L R_1^2}{R_1^2+\omega^2 L^2}.$$

Equating real and imaginary parts separately, we notice that the real parts have only n as a free variable, so we set this first,

$$\frac{R_2}{n^2} \;=\; \frac{\omega^2 L^2 R_1}{R_1^2+\omega^2 L^2} \qquad \Longrightarrow \qquad n=\sqrt{\frac{R_1^2+\omega^2 L^2}{\omega^2 L^2 R_1/R_2}}.$$

With n set, it is just C that is free to set the imaginary part of the load impedance,

$$\frac{1}{n^2\omega C} = \frac{\omega L R_1^2}{R_1^2+\omega^2 L^2} \qquad \Longrightarrow \qquad C=\frac{R_1^2+\omega^2L^2}{n^2\omega^2 L R_1^2}.$$

To be the ideal solution, one should try to express each of the two sought quantities in terms only of the known ones. The above expression for C requires a solution of n, so we can substitute the expression for n into it. This results in a large simplification,

$$C \;=\; \frac{R_1^2+\omega^2 L^2}{\frac{R_1^2+\omega^2 L^2}{\omega^2 L^2 R_1/R_2}\,\omega^2 L R_1^2} \;=\; \frac{L}{R_1 R_2}.$$

It's nice if you did that, but as we didn't say absolutely clearly that each separate expression in the solution shall not depend on the other, we won't deduct any points for leaving C in terms of n.

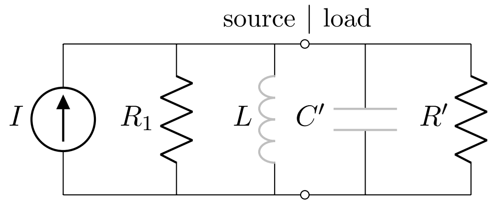
A note about choices of 'source' and 'load'. We made probably the most obvious choice, by including the transformer in the load as its value n was one of the free variables. But the dividing line between source and load doesn't matter as long as one does not include any components in the 'load' that can consume or produce active power: remember that the maximum power theorem is about maximising active power to the load impedance, so if the original task is to maximise the active power to a particular component or set of components then the 'load impedance' chosen for the solution must have the same active power as those components. The transformer, capacitor and inductor cannot consume or produce any active power, so they could be included either in the load or in the source, and the condition $Z_\mathrm{l} = Z_\mathrm{s}^*$ would still be valid.

b) What is the value of this maximum power to R_2?

A long way to approach this is to solve the whole circuit with the chosen values of n and C from part 'a'. It's not very recommended. After much work, it should reduce to the expression below.

A shorter way is to consider that the maximum power is a property of the source. If we know that the load is chosen to extract the maximum power from the source, then we don't need to consider the details of the load any more, but just to find the source's maximum power. That could be done for example by studying the case with the simplest possible form of load that is the complex conjugate of the source's impedance.

With the definition of 'source' that we chose in 'a', that simplest load would be a parallel combination of $R' = R_1$ and a capacitor C' that 'cancels' L by having $\omega^2 LC' = 1$. Then the capacitor and inductor in parallel become an infinite impedance (open circuit), and so half the short-circuit current of the source passes in the load resistor.



Thus, since the short-circuit current is the current-source current,

$$P_{\max} = \frac{I^2 R_1}{4}.$$

Notice that it would be even simpler if we moved the inductor to be part of the 'load' for finding the source's maximum power: that is valid, as the inductor does not consume or produce active power.

Q9.

Angular frequency ω.

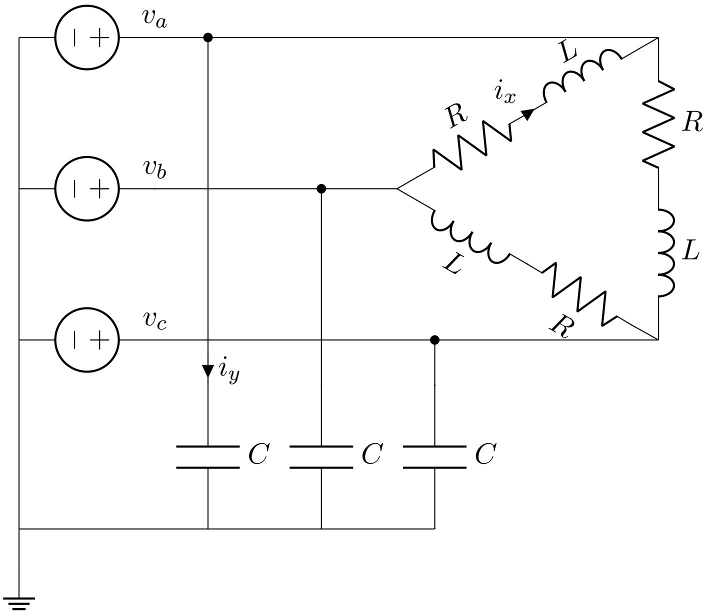
Phase-rotation a,b,c.

Line-voltage U.

Reference $\angle v_a = 0$.

Thus,

$$v_a = \frac{U}{\sqrt{3}}\angle 0$$
$$v_b = \frac{U}{\sqrt{3}}\angle \frac{-2\pi}{3}$$
$$v_c = \frac{U}{\sqrt{3}}\angle \frac{2\pi}{3}$$



a) What apparent power is supplied by the source?

The only things here apart from the source are the impedances: three types, three of each. The complex power that these consume must be the complex power that the source produces, and similiarly therefore for any quantity derived from complex power, such as apparent power.

In each impedance the voltage is fixed by the source, so its complex power is easily found from the relation $|u|^2/Z^*$ for a voltage u applied to impedance Z. The total is

$$S = 3\frac{\left(U/\sqrt{3}\right)^2}{\left(\frac{1}{\mathrm{j}\omega C}\right)^*} + 3\frac{U^2}{(R+\mathrm{j}\omega L)^*} = U^2\left(\frac{3R}{R^2+\omega^2L^2} + \mathrm{j}\frac{3\omega L}{R^2+\omega^2L^2} - \mathrm{j}\omega C\right).$$

Note the importance of adding powers as complex powers, not apparent powers: for example, the capacitor and inductor cancel each other to some extent, but the sums of their apparent powers would simply add.

The question was about the apparent power, so we must now take the absolute value,

$$|S| = U^2\sqrt{\left(\frac{3R}{R^2+\omega^2L^2}\right)^2 + \left(\frac{3\omega L}{R^2+\omega^2L^2} - \omega C\right)^2}.$$

b) What is i_x as a phasor (magnitude and angle)?

By KVL and Ohm's law,

$$i_x = \frac{v_b - v_a}{R+\mathrm{j}\omega L}.$$

Putting in the phasor values of v_a and v_b, and simplifying,

$$i_x = \frac{\frac{U}{\sqrt{3}}\left(1\angle\frac{-2\pi}{3} - 1\angle 0\right)}{R+\mathrm{j}\omega L} = \frac{\frac{U}{\sqrt{3}}\left(\cos\frac{-2\pi}{3} - 1 + \mathrm{j}\sin\frac{-2\pi}{3}\right)}{R+\mathrm{j}\omega L} = \frac{\frac{U}{\sqrt{3}}\left(\frac{-1}{2} - 1 + \mathrm{j}\frac{-\sqrt{3}}{2}\right)}{R+\mathrm{j}\omega L} = \frac{\frac{U}{\sqrt{3}}\left(\frac{-3}{2} - \mathrm{j}\frac{\sqrt{3}}{2}\right)}{R+\mathrm{j}\omega L}.$$

In polar form this is

$$i_x = \frac{U\left(-\frac{\sqrt{3}}{2} - \mathrm{j}\frac{1}{2}\right)}{R+\mathrm{j}\omega L} = \frac{U\angle \operatorname{atan}\frac{1}{\sqrt{3}} - \pi}{R+\mathrm{j}\omega L} = \frac{U\angle\frac{-5\pi}{6}}{R+\mathrm{j}\omega L} = \frac{U}{\sqrt{R^2+\omega^2L^2}}\angle \operatorname{atan}\frac{-\omega L}{R} - \frac{5\pi}{6}.$$

The $-\pi$ after the atan was added because the real part of the complex number was negative; it could alternatively have been $+\pi$. The result, of $-5\pi/6$ radians or $-150°$, could be seen by drawing the phasor diagram, bearing in mind the usual relation of line-voltages having 30° shifts from phase-voltages.

c) What value of capacitance C is needed in order for the source to supply purely active power?

In part 'a' we found an expression for the total complex power supplied by the source. In order for the source to supply purely active (real) power, the reactive power must be zero. The imaginary part of the complex power expression is zero if

$$C = \frac{3L}{R^2 + \omega^2 L^2}.$$

d) The top phase of the three-phase voltage source 'disappears', becoming an open-circuit: find i_y.

In contrast to the earlier parts of this question, this is now an *unbalanced* three-phase system. To aid thinking, it's worth drawing the diagram with the top source removed, to see that i_y is all coming through two parts of the delta load.

One way. If you're really confident about three-phase systems, phasor diagrams, Thevenin equivalents, spatial thinking, symmetry, etc, it might be possible to see a quick intuitive approach. Consider the Thevenin source seen by the capacitor in which i_y is marked. One terminal is at zero potential. The other terminal is in the middle of a voltage divider between two equal impedances of $R + \mathrm{j}\omega L$, that are connected between potentials v_b and v_c; its potential is therefore halfway between those potentials v_a and v_b in the complex plane. The Thevenin impedance is the parallel sum of the two equal impedances.

$$U_{\mathrm{T}} = \frac{-1}{2} \cdot \frac{U}{\sqrt{3}} \qquad \text{and} \qquad Z_{\mathrm{T}} = \frac{R + \mathrm{j}\omega L}{2}.$$

The current i_y is what this Thevenin source would supply to a capacitor C, which is

$$i_y \;=\; \frac{-U}{2\sqrt{3}} \cdot \frac{1}{\frac{1}{2}R + \mathrm{j}\omega\frac{1}{2}L - \mathrm{j}\frac{1}{\omega C}} \;=\; \frac{U}{\sqrt{3}\sqrt{R^2 + \left(\omega L - \frac{2}{\omega C}\right)^2}} \angle \pi - \operatorname{atan}\frac{\omega L - \frac{2}{\omega C}}{R}.$$

Another way. It's likely you'd feel more confident doing it the more 'formal' way. We'll define potential v_x at the node where i_y is marked; that node previously was marked v_a, but it could be confusing to have different meaning for v_a in different sub-questions. This node has three branches connected to it: they are all impedances, connecting to known potentials. By KCL at this node, v_x,

$$0 \;=\; \frac{v_x - 0}{\frac{1}{\mathrm{j}\omega C}} + \frac{v_x - v_b}{R + \mathrm{j}\omega L} + \frac{v_x - v_c}{R + \mathrm{j}\omega L} \;=\; v_x\, \mathrm{j}\omega C\,(R + \mathrm{j}\omega L) + v_x - v_b + v_x - v_c,$$

resulting in

$$v_x \;=\; \frac{v_b + v_c}{2 + \mathrm{j}\omega C\,(R + \mathrm{j}\omega L)} \;=\; \frac{v_b + v_c}{2 - \omega^2 CL + \mathrm{j}\omega CR}$$

The current in the capacitor is then directly found as

$$i_y \;=\; \frac{v_x - 0}{\frac{1}{\mathrm{j}\omega C}} \;=\; \frac{\mathrm{j}\omega C\,(v_b + v_c)}{2 + \mathrm{j}\omega C\,(R + \mathrm{j}\omega L)} \;=\; \frac{v_b + v_c}{\frac{2}{\mathrm{j}\omega C} + (R + \mathrm{j}\omega L)}.$$

The sum $v_b + v_c$ is $\frac{U}{\sqrt{3}}\left(1\angle -120° + 1\angle +120°\right)$, which reduces to $\frac{-U}{\sqrt{3}}$.

Substituting this, and making into polar form,

$$i_y \;=\; \frac{\frac{U}{\sqrt{3}}\angle\pi}{R + \mathrm{j}\omega L - \mathrm{j}\frac{2}{\omega C}} \;=\; \frac{U}{\sqrt{3}\,\sqrt{R^2 + \left(\omega L - \frac{2}{\omega C}\right)^2}} \angle \pi - \operatorname{atan}\frac{\omega L - \frac{2}{\omega C}}{R}.$$