Permitted material: Beyond writing-equipment, up to three pieces of paper up to A4 size can be brought, with free choice of content: handwritten, printed; small, large; text, diagram, image; one or both sides, etc. These papers do not need to be handed in with the exam.

Unless it is stated otherwise, the final answer to a question should be expressed in terms of the known quantities given in the question, and any clear simplifications should be done. Component values such as R for a resistor, U for an independent voltage source, or K for a dependent source, are assumed to be known quantities. Marked currents or voltages such as i_x are assumed to be definitions, not known quantities.

Clearly drawn and labelled diagrams are a good way to help yourself avoid mistakes, and to make clear to others what you are doing. By showing clearly your intermediate steps in a solution, you improve your chance of getting points even if the final result is wrong. You may write in Swedish or English; but we suggest that writing in either is seldom necessary if you make good use of diagrams and equations!

Determination of exam grade. Denote as A, B and C the available points from sections A, B and C of this exam: A=12, B=10, C=18. Denote as a, b and c the points actually obtained in the respective sections, and as a_k and b_k the points från KS1 and KS2, and as h the homework 'bonus'. The requirement for passing the exam (E or higher) is:

$$\frac{\max(a, a_k)}{A} \geq 40\% \quad \& \quad \frac{\max(b, b_k)}{B} \geq 40\% \quad \& \quad \frac{c}{C} \geq 40\% \quad \& \quad \frac{\max(a, a_k) + \max(b, b_k) + c + h}{A + B + C} \geq 50\%$$

The grade is then determined by the total including bonus, i.e. the last of the terms above: boundaries (%) are 50 (E), 60 (D), 70 (C), 80 (B), 90 (A). If the exam misses a pass by a small margin on just one criterion, a grade of Fx may be registered, with the possibility of completing to E by an extra task arranged later.

For this VT19 round, students who have their final project-task approved will get full points on Q9 in this exam.
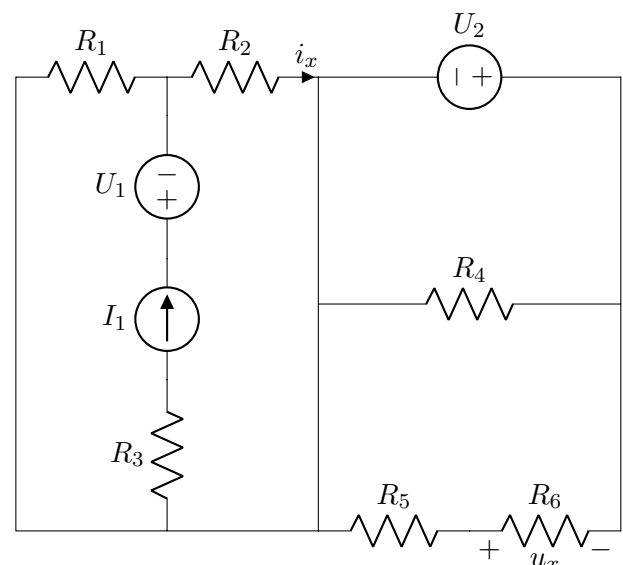
<div style="text-align: right">Nathaniel Taylor (08 790 6222)</div>

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Section A. Direct Current

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1) [4p]

Find:

a) [1p]   the voltage u_x

b) [1p]   the current i_x

c) [1p]   the power delivered from source I_1

d) [1p]   the power delivered from source U_2

2) [4p]

Write equations that could be solved to find the potential v_0 in terms of the component values.

You do not have to simplify or solve the equations. The equations should be sufficient to give a solution without needing further information.
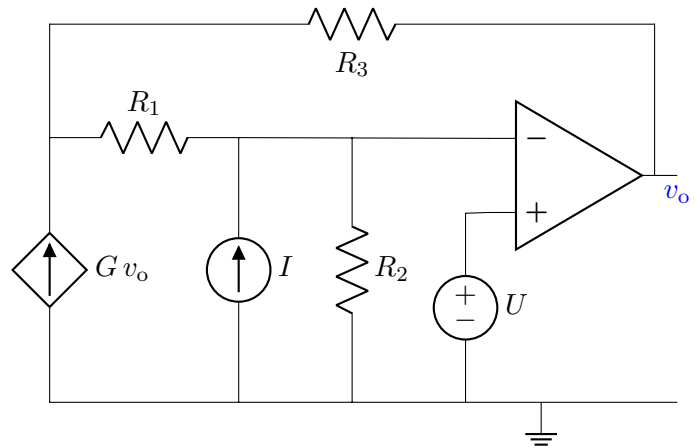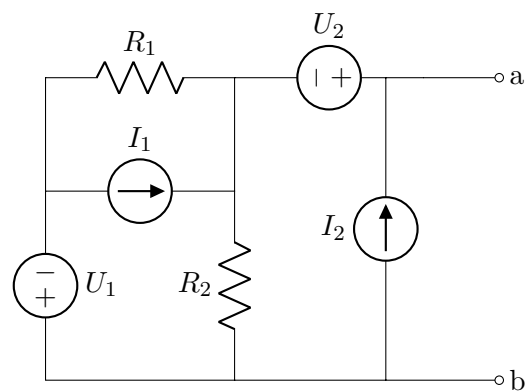

3) [4p]

In this circuit,

$$U_1 = U_2 = U, \quad R_1 = R_2 = R, \quad I_1 = I_2 = U/R.$$

The final answers should be in terms of U and/or R and simplified; they should *not* include the other names such as U_1 etc.

a) [2p]   What value of resistor, connected between terminals a-b, will maximize the power delivered by this circuit to the resistor.

b) [2p]   A voltage source is now connected at a-b instead of a resistor, with its + pole to terminal 'a'. What value should the voltage source have in order to maximize the power delivered to it.

Section B.   Transient Calculations
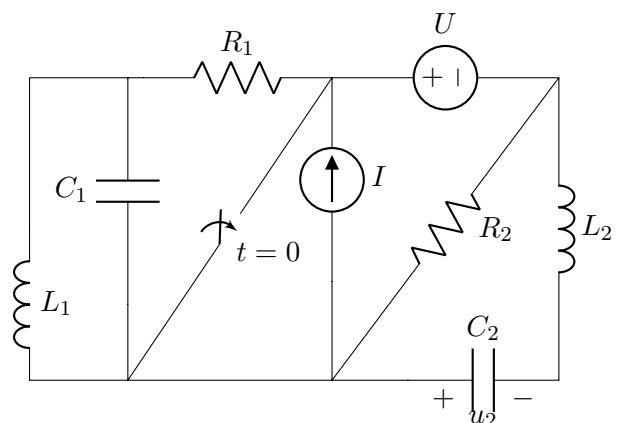
4) [5p]   Find:

a) [1p]   Power absorbed by R_1 at $t = 0^-$

b) [1p]   Energy stored in C_2 at $t = 0^-$

c) [1p]   Energy stored in L_2 at $t = 0^+$

d) [1p]   Power absorbed by R_2 at $t = 0^+$

e) [1p]   Rate of change $\dfrac{\mathrm{d}u_2}{\mathrm{d}t}$ at $t = 0^+$
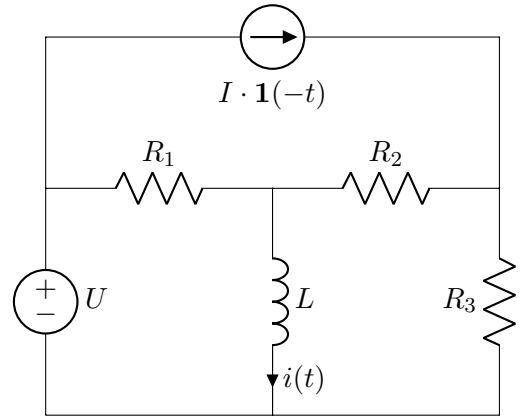
5) [5p]

a) [4p] Find the current $i(t)$, for $t > 0$.

b) [1p] Find the power absorbed by R_1 for $t > 0$.

Both of the above are expected to be functions of time.
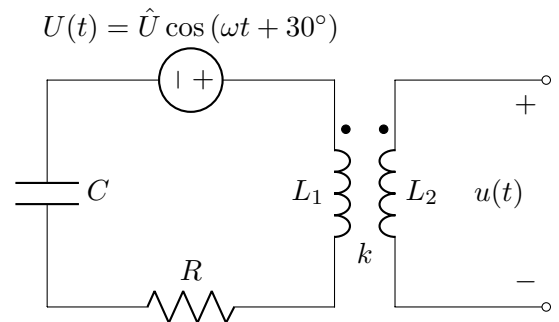

Section C.   Alternating Current

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6) [4p]

Determine $u(t)$.

Steady-state sinusoidal conditions can be assumed.

The two inductors have some mutual inductance, described in terms of coupling coefficient k.

Recall that k is a dimensionless coefficient, not a mutual inductance!

$$U(t) = \hat{U} \cos\left(\omega t + 30°\right)$$

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7) [4p]

a) [2p] Determine this circuit's network function,

$$\frac{u(\omega)}{U(\omega)}.$$
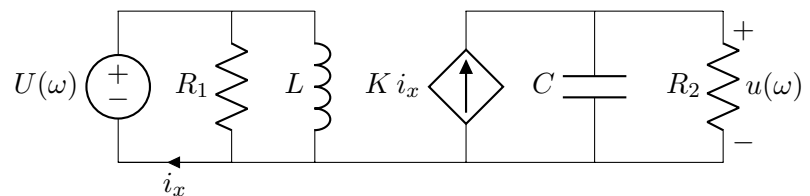
b) [2p] Sketch a Bode amplitude plot of the following network function:

$$H(\omega) = \frac{K\left(1 + \mathrm{j}\omega/\omega_1\right)}{\left(1 + \mathrm{j}\omega/\omega_2\right)\left(1 + \mathrm{j}\omega/\omega_3\right)^2}.$$

(This is *not* the same as the function in part 'a'!)

Assume $K = \sqrt{10}$ and $\omega_1 \ll \omega_2 \ll \omega_3$.
Mark the frequencies ω_1 etc, any non-zero gradients, and any known amplitudes.

Don't be worried by the squared term: just write the term twice if that helps.

8) [4p]

The current source has angular frequency ω.
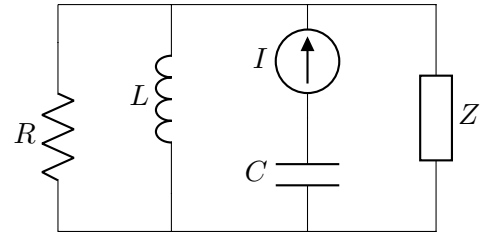
a) [2p]   What value of impedance Z will maximize the active power delivered to this impedance? Express this in terms of other component values.

b) [2p]   What is the value of the active power into Z in the situation described in part 'a)'.

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9) [6p]

At the left of this circuit is a balanced three-phase source consisting of three single-phase voltage sources. It has *line*-voltage U, angular frequency ω, and phase-rotation 1,2,3. The phase of u_1 is taken as the reference: $\angle u_1 = 0$.
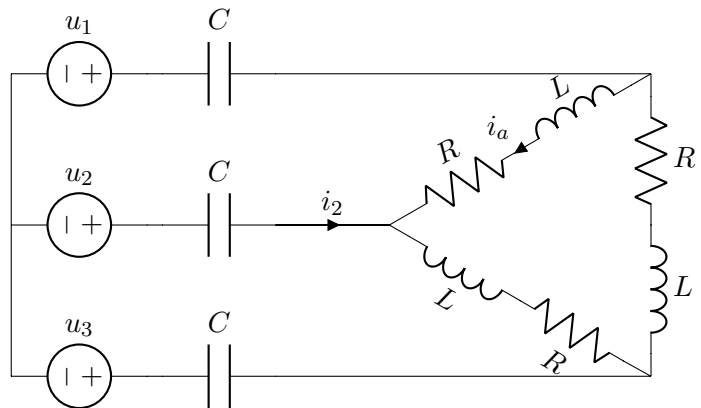
a) [2p]   Determine the complex power supplied by the source.

b) [2p]   What value of capacitance C is needed in order for the source to supply zero reactive power?

c) [1p]   Determine i_2 as a phasor (magnitude and angle).

d) [1p]   Phase '2' of the source melts. In its new state the circuit can be modelled by replacing the middle voltage-source in the diagram by an open-circuit. What now is i_a (magnitude and angle)?

The End.   *Please don't waste remaining time ... check your solutions!*

Översättningar:

Hjälpmedel: Upp till tre A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text, diagram, bild; stor eller liten textstorlek, o.s.v. Dessa måste inte lämnas in med skrivningarna.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, K för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas.

Var tydlig med diagram och definitioner av variabler. Du får skriva på svenska eller engelska, men vi rekommenderar att diagram och ekvationer används i stället i de flesta fall.

1. [4p] Bestäm följande:
a) [1p] spänningen u_x
b) [1p] strömmen i_x
c) [1p] effekten levererad från källan I_1
d) [1p] effekten levererad från källan U_2

2. [4p] Skriv ekvationer som skulle kunna lösas, utan vidare information, för att bestämma potentialen v_0 som funktion av kretsens komponentvärden. Du måste inte förenkla eller lösa ekvationerna.

3. [4p] Vilket komponentvärde behövs för att maximaleffekt ska levereras när komponenten kopplas mellan a-b:
a) [2p] motstånd
b) [2p] spänningskälla med +-polen kopplad till 'a'
De slutliga uttrycken borde innehålla storheterna U och/eller R och förenklad; storheterna U_1 o.s.v. borde inte vara kvar.

4. [5p] Bestäm:
a) [1p] Effekten absorberad av R_1 vid $t = 0^-$.
b) [1p] Energin lagrad i C_2 vid $t = 0^-$.
c) [1p] Energin lagrad i L_2 vid $t = 0^+$.
d) [1p] Effekten absorberad av R_2 vid $t = 0^+$.
e) [1p] Tidsderivaten av spänning, $\frac{\mathrm{d}u_2}{\mathrm{d}t}$ vid $t = 0^+$.

5. [5p]
a) [4p] Bestäm strömmen $i(t)$ vid $t > 0$.
b) [1p] Bestäm effekten absorberad av R_1 vid $t > 0$.

6. [4p] Bestäm $u(t)$. Använd jω-metoden. Obs att k inte är ömsesidiginduktans utan kopplingsfaktor.

7. [4p]
a) [2p] Härled kretsens nätverksfunktion, $u(\omega)/U(\omega)$.
b) [2p] Skissa ett Bodeamplituddiagram av $H(\omega) = \frac{K(1+\mathrm{j}\omega/\omega_1)}{(1+\mathrm{j}\omega/\omega_2)\,(1+\mathrm{j}\omega/\omega_3)^2}$.
Obs. kvadrat på en term i nämnaren. Den här är inte samma funktion som i 'a'!
Antag $K = \sqrt{10}$ och $\omega_1 \ll \omega_2 \ll \omega_3$. Markera viktiga punkter och lutningar.

8. [4p] Källan har vinkelfrekvens ω.
a) [2p] Bestäm värden Z som ger maximaleffekt till impedansen Z.
b) [2p] Hur mycket effekt blir den till Z vid situationen från delta 'a'?

9. [6p] Till vänster i kretsen är en balanserad trefas källa, med huvudspänning U, vinkelfrekvens ω, och fasföljd 1,2,3. Fasvinkeln av u_1 tas som referens, d.v.s. $\underline{/u_1} = 0$.
a) [2p] Vilken komplexeffekt levererar källan?
b) [2p] Vilken kapacitans C behövs för att källan inte försörjer någon reaktiveffekt.
c) [1p] Bestäm i_2 (som fasvektor – magnitud och vinkel).
d) [1p] Fas 2 i källan smälter. Situationen efteråt kan modelleras genom att ersätta spänningskällan i mitten (u_2) med en öppenkrets. Bestäm i så fall i_a (magnitud och vinkel).

Solutions (EI1120 TEN1 VT19, 2019-06-07)

Having given very long, thorough answers to the ordinary exam this year (March 2019), we try this time to please those who prefer concise answers.

Q1.

a) $\quad u_x = \dfrac{-R_6}{R_5 + R_6} U_2$

KVL to find voltage across R_5 and R_6, then voltage division.

b) $\quad i_x = \dfrac{R_1}{R_1 + R_2} I_1$

Current division between R_1 and R_2. KCL around U_1 shows that the current I_1 must split between these two resistors. Note that these resistors are in parallel, as they both connect between the same pair of nodes.

c) $\quad P_{I_1} = U_1 I_1 + \left(\dfrac{R_1 R_2}{R_1 + R_2} + R_3 \right) I_1^2$

KVL to find the the voltage across I_1, then multiply this voltage by I_1. Careful to get the right directions for power *from* this source. It helps to simplify by combining R_1 and R_2 in parallel.

d) $\quad P_{U_2} = U_2^2 \left(\dfrac{1}{R_4} + \dfrac{1}{R_5 + R_6} \right) = \dfrac{U_2^2 (R_4 + R_5 + R_6)}{R_4 (R_5 + R_6)}$

This is U_2^2 / R_{eq} where R_{eq} is the equivalent of R_4 in parallel with the series pair $R_5 + R_6$. A cleaner way to do it may be just KCL at the right of U_2, after using KCL and Ohm's law to find the currents in the two resistor branches. (If doing KCL on the left of U_2, one would have to notice that i_x coming into this node must be the same as the current going out of this node towards R_3, so this current cancels itself in the KCL).

Q2.

Extended nodal analysis

Define the node at the left of R_1 as potential v_1, the node above R_2 as v_2, and the node above U as v_3. The only other nodes are the reference node and the node already marked v_o. In the KCL we'd need currents in the voltage sources, which are U and the opamp output. Define i_α into the + terminal of source U, and i_β out of the opamp output.

Write KCL for all nodes except the reference.

$$\text{KCL(0):} \quad 0 = -i_\beta + \frac{v_\mathrm{o} - v_1}{R_3} \tag{1}$$

$$\text{KCL(1):} \quad 0 = \frac{v_1 - v_\mathrm{o}}{R_3} + \frac{v_1 - v_2}{R_1} - G v_\mathrm{o} \tag{2}$$

$$\text{KCL(2):} \quad 0 = \frac{v_2 - v_1}{R_1} - I + \frac{v_2}{R_2} + 0 \tag{3}$$

$$\text{KCL(3):} \quad 0 = i_\alpha + 0 \tag{4}$$

The above are 4 equations, in 6 unknowns. The two voltage sources have given us the problem of two unknown currents,[1] and the same two sources solve the problem by telling us further relationships about voltage.

$$v_3 = U \tag{5}$$

$$v_2 = v_3 \qquad \text{ideal opamp with negative feedback} \tag{6}$$

---

[1]You might think i_α isn't unknown, as it's obviously zero; but that 'obvious' point is what KCL(3) is telling us, so if we treat i_α as known we also treat KCL(3) as having no further useful information, in which case we've 3 equations in 5 unknowns.

The above equations (1)–(6) are a sufficient solution.

Nodal analysis: simplify on the way, e.g. supernode

The nodes with potentials that were called v_o and v_3 in the method above can now be ignored for KCL, as they are both treated as part of the 'ground supernode'. The symbol v_3 is not needed, as we can immediately call this potential $-U$. The symbol v_o is still needed, as it appears in another KCL and is anyway the quantity we're wanting to find from our equations! Two KCL equations remain, in three unknowns.

$$\text{KCL(1):} \quad 0 \quad = \quad \frac{v_1 - v_o}{R_3} + \frac{v_1 - v_2}{R_1} - G v_o \tag{1}$$

$$\text{KCL(2):} \quad 0 \quad = \quad \frac{v_2 - v_1}{R_1} - I + \frac{v_2}{R_2} + 0 \tag{2}$$

$$\tag{3}$$

The further equation that is needed is given by the opamp, which was what also gave the further unknown v_o. As above, this is the relation saying that the two inputs of an ideal opamp with negative feedback have equal potential,

$$v_2 \quad = \quad U. \tag{4}$$

Q3.

These questions are about maximum power. Recall from the theory, that maximum power from a linear two-terminal dc circuit is obtained when the voltage at the terminals is half of its open-circuit value (which is also when the output current is half of its short-circuit value), and that one way of acheiving this is by a load of a resistor equal to the circuit's Thevenin resistance.

a) $\quad \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{2}R$

As stated above, the maximum power condition for a resistive load is when the load resistance equals the circuit's Thevenin resistance.

We can find this circuit's Thevenin resistance between a-b by setting the (independent) sources to zero: voltage sources are short circuits, and current sources are open circuits. Then the two resistors R_1 and R_2 are in parallel between the terminals, leading to the result $R/2$.

b) $\quad \frac{1}{2} \left((I_1 + I_2) \frac{R_1 R_2}{R_1 + R_2} - U_1 \frac{R_2}{R_1 + R_2} + U_2 \right) = \frac{3}{4}U \qquad$ +-terminal to 'a'.

As also stated above, the maximum power condition is when the terminal voltage is half of its open-circuit value. So we need to connect a voltage source whose value is half of the circuit's open-circuit (Thevenin) voltage.

Two efficient ways to find this circuit's Thevenin voltage are superposition for each source at a time, or nodal analysis with supernodes, which results in just one KCL equation.

Another way is repeated source-transformation. The pair R_1 and I_1 are parallel so they form a Norton source that can be replaced with its equivalent Thevenin source of $I_1 R_1$ and R_1 in series: the question tells us that all current sources had value U/R and all resistors had value R, so we can immediately write this Thevenin source as U in series with R. After re-drawing the circuit, the new Thevenin source is in series with U_1 and has its voltage in the oppostive direction, so those voltages cancel: the three components U_1, I_1, R_1 therefore reduce to just R. This R is in parallel with R_2, so together they form a resistor $R/2$, which is the equivalent for all four leftmost components of the original circuit. The open-circuit voltage between a-b can then be found by KVL passing through the $R/2$ resistor (in which all of I_2 must flow downwards when a-b is open-circuit) and then through the source U_2.
Thus: $u_{ab(\text{oc})} = \frac{R}{2}\frac{U}{R} + U = \frac{3}{2}U$.
This value is halved to find value of the external voltage-source that will extract the maximum power from the shown circuit's terminals a-b.

Q4.

At $t = 0^-$
Equilibrium. Switch open. The circuit can in this case be drawn as shown on the right.

It can be directly seen that: $u_{C1} = 0$ and $i_{L2} = 0$.

Ignoring the open-circuit branches, this circuit has three parallel branches between the top and bottom nodes. It is a good case for nodal analysis or superposition.

By superposition, $i_{L1} = \frac{U}{R_1 + R_2} + \frac{IR_2}{R_1 + R_2} = \frac{U + IR_2}{R_1 + R_2}$.

By KCL, the current down through R_2 is $I - i_{L1}$.

So, by Ohm's law and KVL, $u_{C2} = (I - i_{L1}) R_2$, which simplifies to $u_{C2} = \frac{IR_1 - U}{R_1 + R_2} R_2$.

From the above we have the continuous variables that could be useful for $t = 0^+$, and we now can answer the questions about $t = 0^-$:

$$\textbf{a)}\ P_{R1}(0^-) = \left(\frac{U + IR_2}{R_1 + R_2} \right)^2 R_1, \qquad \textbf{b)}\ W_{C2}(0^-) = \frac{1}{2} C_2 \left(\frac{IR_1R_2 - UR_2}{R_1 + R_2} \right)^2.$$

At $t = 0^+$
Continuity from the previous equilibrium. The current source is shorted by the switch.

The continuous variables of the four energy-storing components can be represented as sources, whose values are valid just for this instant when the switch has closed. The voltage source of zero (C_1) and the current source of zero (L_2) are more clearly written as a short and open circuit.
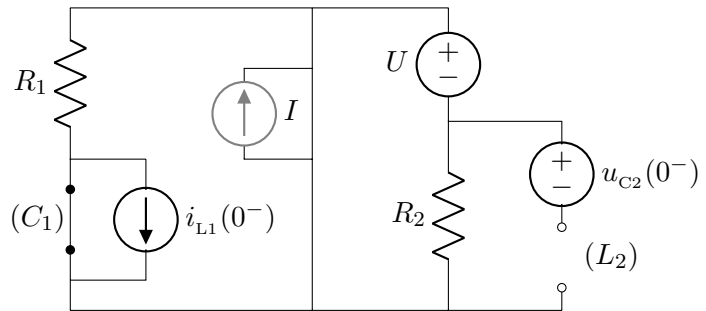
From this rewritten diagram for $t = 0^+$ we can answer the remaining questions.

The stored energy in L_2 at $t = 0^+$ must be the same as at $t = 0^-$. We already saw that the current in L_2 was zero then, so its energy is zero.

$$\textbf{c)}\ W_{L2}(0^+) = 0.$$

The current in R_2 is different now, as the current source is short-circuited by the closed switch. By KVL around the loop of U, R_2 and the switch, the full voltage U is across R_2.

$$\textbf{d)}\ P_{R2}(0^+) = \frac{U^2}{R_2}.$$

The quantity u_2 is marked across C_2 in the original circuit. It is in the opposite direction from the u_{C2} that we marked. In this case it doesn't matter about the direction, as the answer is zero.
This follows because continuity of the current in L_2 holds the current in C_2 to zero at $t = 0^+$, and a capacitor has the relation $i = C \frac{\mathrm{d}u}{\mathrm{d}t}$.

$$\textbf{e)}\ \frac{\mathrm{d}u_2(0^+)}{\mathrm{d}t} = 0$$

Q5.

Before $t = 0$ the current source has current I. From $t = 0$ onwards, it has zero current, so it can be treated as an open circuit.

The marked current $i(t)$ is the continuous variable of the inductor, so we'll solve for this.

Initial condition. At $t = 0^-$ there is equilibrium with the current source active. The inductor, in equilibrium, has zero voltage so it can be treated as a short circuit. The current can then be found by KVL and Ohm's law in U and R_1, plus current division of I between R_2 and R_3. By continuity, this current is the same just after the current-source changes. Hint: draw the circuit carefully for the equilibrium state, in order to be sure of this.

$$i(0^+) = i(0^-) = \frac{U}{R_1} + \frac{IR_3}{R_2 + R_3}.$$

Final state. As $t \to \infty$, we again consider an equilibrium, so the inductor is treated as a short-circuit. This equilibrium is different in that the current source is an open circuit.

$$i(\infty) = \frac{U}{R_1}.$$

Time-constant. During the time $t > 0$, the Thevenin equivalent of the rest of the circuit seen from the inductor's terminals has a resistance of the parallel combination of R_1 and the series-branch $R_2 + R_3$.

$$R_{\mathrm{T}} = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}.$$

The circuit's time-constant is therefore $\tau = L/R_{\mathrm{T}}$.

We know this is a first-order circuit (just one capacitor or inductor within a linear dc circuit), so its response is a decaying exponential to the new equilibrium.

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right) e^{-t/\tau}.$$

Putting the above three values together,

$$i(t) = \frac{U}{R_1} + \frac{IR_3}{R_2 + R_3} e^{-t \frac{R_1 (R_2 + R_3)}{L(R_1 + R_2 + R_3)}} \qquad (t > 0).$$

a) The solution here is $i(t)$ as given above.

b) To answer this, we need somehow to find the voltage across the inductor. This voltage determines the voltage across R_1. As an alternative view, the current that flows in R_2 contributes to the current in R_1, and depends on the inductor voltage.

Define $u(t)$ of the top relative to the bottom of the inductor. Then:

$$u(t) = L \frac{\mathrm{d}i(t)}{\mathrm{d}t} = L \cdot \frac{-R_1 (R_2 + R_3)}{L (R_1 + R_2 + R_3)} \cdot \frac{IR_3}{R_2 + R_3} e^{-t \frac{R_1 (R_2 + R_3)}{L(R_1 + R_2 + R_3)}} = -I \frac{R_1 R_3}{R_1 + R_2 + R_3} e^{-t \frac{R_1 (R_2 + R_3)}{L(R_1 + R_2 + R_3)}}.$$

The power in R_1 is then:

$$P_{\mathrm{R1}} = \frac{(U - u(t))^2}{R_1} = \frac{\left(U + I \frac{R_1 R_3}{R_1 + R_2 + R_3} e^{-t \frac{R_1 (R_2 + R_3)}{L(R_1 + R_2 + R_3)}}\right)^2}{R_1} \qquad (t > 0).$$

Q6.

In this question the coupling of the coupled inductors is described in terms of their coupling coefficient k. The mutual inductance that we usually work with in the circuit equations is found from the relation $M = k\sqrt{L_1 L_2}$.

The 'secondary' of the coupled inductors (L_2) is open-circuit: it therefore has no current, so there is no contribution of the mutual inductance term $M\frac{\mathrm{d}i_2}{\mathrm{d}t}$ for the voltage across L_1. That makes the solution simple, as we can analyse the left part of the circuit independently of L_2.

We'll use ac analysis. Let's take peak-value reference and $\cos(\omega t + 30°)$ as our angle reference: so we describe the source as a phasor $U(\omega) = \hat{U}\underline{/0}$.

Then the current clockwise around the left loop is

$$i(\omega) \;=\; \frac{U(\omega)}{R + \mathrm{j}\omega L_1 + \frac{1}{\mathrm{j}\omega C}} \;=\; \frac{\hat{U}}{R + \mathrm{j}\left(\omega L_1 - \frac{1}{\omega C}\right)}.$$

This current flows into the dotted side of L_1. No current flows in L_2. Therefore, by the equations of mutual inductors, the marked voltage is

$$u(\omega) \;=\; \mathrm{j}\omega L_2 \cdot 0 + \mathrm{j}\omega M i(\omega).$$

In terms of given quantities,

$$u(\omega) \;=\; \frac{\mathrm{j}\omega k\sqrt{L_1 L_2}\hat{U}}{R + \mathrm{j}\left(\omega L_1 - \frac{1}{\omega C}\right)} \;=\; \frac{\omega k\sqrt{L_1 L_2}\hat{U}}{-\mathrm{j}R + \left(\omega L_1 - \frac{1}{\omega C}\right)}.$$

In order to find the corresponding time-function, the magnitude and angle are needed.

$$|u(\omega)| \;=\; \frac{\omega k\sqrt{L_1 L_2}\hat{U}}{\sqrt{R^2 + \left(\omega L_1 - \frac{1}{\omega C}\right)^2}}.$$

$$\underline{/u(\omega)} \;=\; -\mathrm{atan}\frac{-R}{\omega L_1 - \frac{1}{\omega C}} \;=\; \mathrm{atan}\frac{R}{\omega L_1 - \frac{1}{\omega C}}.$$

Putting these as the magnitude and additional angle, into the time-function $u(t) = |u|\cos\left(\omega t + 30° + \underline{/u}\right)$,

$$u(t) \;=\; \frac{\omega k\sqrt{L_1 L_2}\hat{U}}{\sqrt{R^2 + \left(\omega L_1 - \frac{1}{\omega C}\right)^2}} \cos\left(\omega t + 30° + \mathrm{atan}\frac{R}{\omega L_1 - \frac{1}{\omega C}}\right).$$

We should also murmur something about checking that $\omega L_1 > \frac{1}{\omega C}$, and adding π to the result of the atan function if it's not.... We could have avoided that issue by not 'simplifying' so far: we could have kept the j in the top of the expression for $u(\omega)$, and thereby have kept R as the real part at the bottom, leading to an alternative form of solution with a cosine of $\omega t + 30° + 90° + \mathrm{atan}\frac{\frac{1}{\omega C} - \omega L}{R}$.

Q7.

a) The two parts of this circuit do not affect each other through the one node that joins them (no current can flow between them). Only a one-way effect happens, with the current i_x in the left part controlling the source in the right part. We can analyse each part in turn.

$$\text{Left: } i_x = U \left(\frac{1}{R_1} + \frac{1}{j\omega L} \right) = U \frac{R_1 + j\omega L}{j\omega L R_1}, \qquad \text{Right: } u = K i_x \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = K i_x \frac{R_2}{1 + j\omega C R_2}.$$

Thus, chaining the two expressions together to eliminate i_x,

$$\frac{u}{U} = K \cdot \frac{R_2}{1 + j\omega C R_2} \cdot \frac{R_1 + j\omega L}{j\omega L R_1} = \frac{K R_2 (R_1 + j\omega L)}{j\omega L R_1 (1 + j\omega C R_2)} = \frac{1 + j\omega L / R_1}{j\omega \frac{L}{K R_2} (1 + j\omega C R_2)}.$$

The final version of the answer was not required to be in a specific form, so any of the above rearrangments, or similar, would be acceptable.

b) The Bode amplitude diagram at the right is an example.

Important details are the marking of gradients (the zero gradients do not require this) and the 10 dB level for $\omega < \omega_1$, which follows from $20 \log_{10} \sqrt{10}$.

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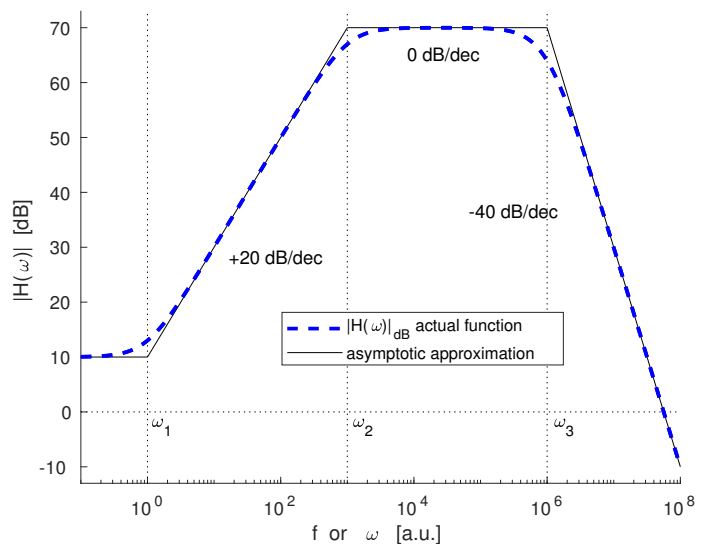
Q8.

The question makes clear that this is a classic ac maximum power situation, where we choose the load Z, and the rest of the circuit is the source.

Considering the source to be everything except the load Z, the impedance of the source at its terminals is the parallel resistor and inductor (the capacitor is not seen, as it is in series with the current source).

a) By the ac maximum power principle, the impedance Z should be chosen as the complex conjugate of this source impedance:

$$Z = \left(\frac{R\,\mathrm{j}\omega L}{R + \mathrm{j}\omega L} \right)^* = \frac{R\,\omega L}{\omega L + \mathrm{j}R}.$$

b) To find the active power to the load in the maximum power condition, it's probably most convenient to consider a load in which Z is formed from a parallel resistor R and capacitor $\frac{1}{\omega^2 L}$. (This parallel pair would have the same impedance as the Z that was found in part 'a'.)

Then we see that the inductor (in the source) and capacitor (in the load) cancel each other's currents. What remains in the circuit is just two parallel resistors R across the current source. The load resistor therefore receives half of the source current. Thus,

$$P_{\mathrm{max}} = \left(\tfrac{1}{2}I \right)^2 R = \tfrac{1}{4}I^2 R.$$

One must be careful to draw a good diagram and think clearly! The above is good if the load and source impedances are parallel connections of resistor and other component, which fits well with thinking of admittance (its real and imaginary part describe the resistor and the other component respectively). On the other hand, an impedance such as the load Z, has real and imaginary parts that describe a *series* resistor and other component. An easy mistake is to think of the real part of impedance as describing the resistor in a parallel model. As in some earlier exam solutions, we here have taken advantage of the fact that maximum power is a feature of the source, so to find the maximum power we can select a load in the form that's most easy to analyse, without having to do it the long way by calculating the load impedance and then further calculating what parallel components would have this impedance.

Q9.

The balanced delta load can with advantage be transformed to a star equivalent, in which each phase has impedance $(R/3 + \mathrm{j}\omega L/3)$. Then each capacitor is in series with a phase of the load. The system is balanced three-phase, so the neutral point of the load has the same potential as the neutral of the source.

In this case we haven't defined any potential, as it's only the voltages that matter for results such as powers, currents etc. The symmetry tells us that the voltage of each phase of the source is the same as the voltage of that same phase of the line and star-connected load.

a) In the transformed (Y-connected load) circuit described above, each phase of the source is connected to a series combination of $\frac{1}{\mathrm{j}\omega C} + R/3 + \mathrm{j}\omega L/3$. Each phase of the source has a voltage magnitude of $U_{\mathrm{p}} = U/\sqrt{3}$. Therefore, for the entire three-phase source, the complex power supplied is

$$S = 3\frac{U_{\mathrm{p}}^2}{Z_{\mathrm{tot}}^*} = \frac{3\left(\frac{U}{\sqrt{3}}\right)^2}{\left(R/3 + \mathrm{j}\omega L/3 + \frac{1}{\mathrm{j}\omega C}\right)^*} = \frac{U^2}{R/3 + \mathrm{j}\left(\frac{1}{\omega C} - \omega L/3\right)} = \frac{3U^2}{R + \mathrm{j}\left(\frac{3}{\omega C} - \omega L\right)}.$$

It's not strictly necessary to convert this to a clean rectangular form here, but later it will anyway be necessary to separate the imaginary part. Let's get it into a neat form now:

$$S = \frac{3U^2}{R^2 + \left(\omega L - \frac{3}{\omega C}\right)^2}\left(R + \mathrm{j}\left(\omega L - \frac{3}{\omega C}\right)\right)$$

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b) The requirement here is to have no reactive power supplied by the source. In other words, the imaginary part of the complex power found in part 'a' should be zero. The condition for this is

$$\frac{3}{\omega C} - \omega L = 0 \quad \Longrightarrow \quad C = \frac{3}{\omega^2 L}.$$

c) This is still a balanced situation of source and load. The above method of a Y-connected load equivalent is still useful.

Looking at just the middle phase (number 2), the voltage u_2 is applied across one of the phases of the load, so

$$i_2 = \frac{u_2}{R/3 + \mathrm{j}\omega L/3 + \frac{1}{\mathrm{j}\omega C}}.$$

In view of the line-voltage, phase-rotation and reference angle, $u_2 = \frac{U}{\sqrt{3}}\angle{-120°}$. Therefore,

$$i_2 = \frac{\frac{U}{\sqrt{3}}\angle{-120°}}{R/3 + \mathrm{j}\omega L/3 + \frac{1}{\mathrm{j}\omega C}} = \frac{\sqrt{3}U\angle{-120°}}{R + \mathrm{j}\omega L + \frac{3}{\mathrm{j}\omega C}} = \frac{\sqrt{3}U}{\sqrt{R^2 + \left(\omega L - \frac{3}{\omega C}\right)^2}}\angle{-120° - \mathrm{atan}\frac{\omega L - \frac{3}{\omega C}}{R}}.$$

d) Now it is an unbalanced case: the source is no longer a balanced three-phase source. The load is still balanced, so the balanced Δ-Y transformation is still valid. However, the marked i_a is in one phase of the original Δ-connected load, and there is no clear advantage to using the transformed load for this solution.

With phase 2 of the source open-circuit, we can rub out this and the middle capacitor. Redrawing, the two further-left phases of the Δ-connected load are now in series, and that series pair is in parallel with the other phase that connects between the top and bottom line. We don't immediately know the voltage across the load phases, as there are capacitors between the source and the load phases.

Let's define $Z_c = \frac{1}{\mathrm{j}\omega C}$ and $Z_\mathrm{p} = R + \mathrm{j}\omega L$. Then the total current passing from source u_1 is

$$i_1 = \frac{u_1 - u_3}{2Z_c + \frac{2Z_\mathrm{p}^2}{2Z_\mathrm{p} + Z_\mathrm{p}}} = \frac{U\angle{30°}}{2Z_c + \frac{2}{3}Z_\mathrm{p}}.$$

What we want is i_a, which is the part of this current i_1 that divides into the left of the two paths through the load, i.e. through the two series-connected phases of the load. By current division this is $\frac{i_1 Z_\mathrm{p}}{2Z_\mathrm{p} + Z_\mathrm{p}} = i_1/3$, which gives:

$$i_a = \frac{i_1}{3} = \frac{U\angle{30°}}{6Z_c + 2Z_\mathrm{p}} = \frac{U\angle{30°}}{\frac{6}{\mathrm{j}\omega C} + 2R + \mathrm{j}2\omega L} = \frac{U\angle{30°}}{2R + \mathrm{j}\left(2\omega L - \frac{6}{\omega C}\right)}.$$

Getting this into polar form,

$$i_a = \frac{U}{\sqrt{4R^2 + \left(2\omega L - \frac{6}{\omega C}\right)^2}}\angle{30° - \mathrm{atan}\frac{\omega L - \frac{3}{\omega C}}{R}}.$$