KTH **EI1120 VT2025 KS1** 2025-01-31 08:00–10:00 N. Taylor

Permitted materials: pens, other writing equipment, calculator, 1 A4 paper with free choice of written/printed content on both sides.

Where specific values are given for the components (e.g. 12 V), final answers should be a number and unit. You do not need to give a detailed description of your working (solution steps), but you should give a few lines to show the principle you've used. Grading is mainly based on the final answer, but some credit can be obtained for a clearly correct method with a minor error.

Pages: 2. Questions: 4. Total available points: 12. This KS can contribute to the final exam, for which 40% is a requirement (in either KS1 or the respective part in the exam) and 50% average is required over this and the other two parts of the exam.

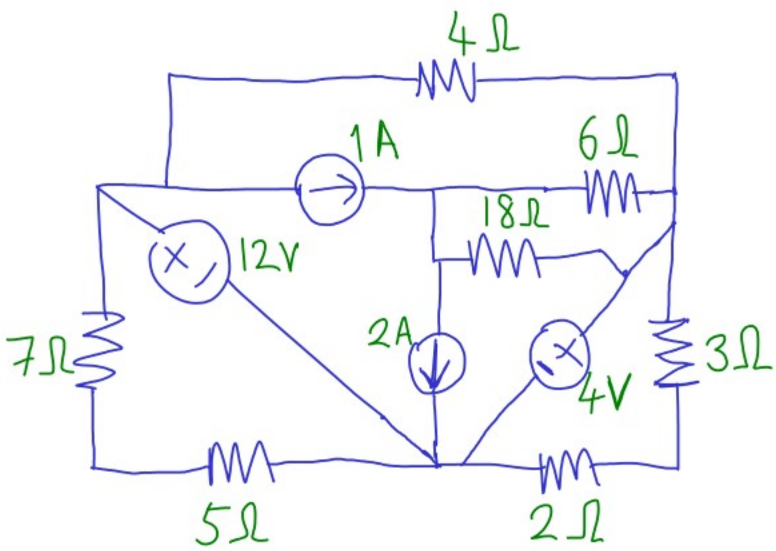
1) [4p]

a. How much power is supplied to the 5 Ω resistor?

b. How much power is supplied to the 4 Ω resistor?

c. How much power is supplied to the 6 Ω resistor?

d. How much power is supplied from the 4 V source?

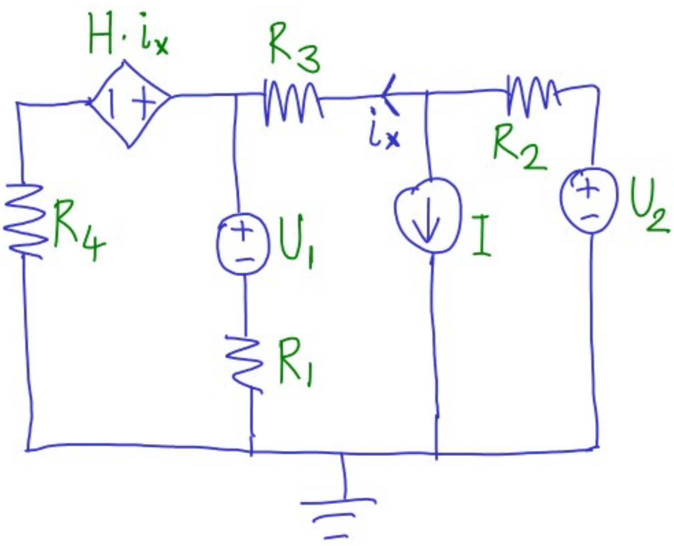


2) [1p] *No explanation is needed for the numeric answers of these questions.*

a. How many nodes has this circuit, including the reference (earth) node?

b. Using 'extended' nodal analysis (wherein no simplifications are made, but a set of equations is written based on a few simple rules), how many equations will be used to represent this circuit?

c. Using the simplifications provided by 'supernodes', how many KCL equations would be written for this circuit?



3) [3p]

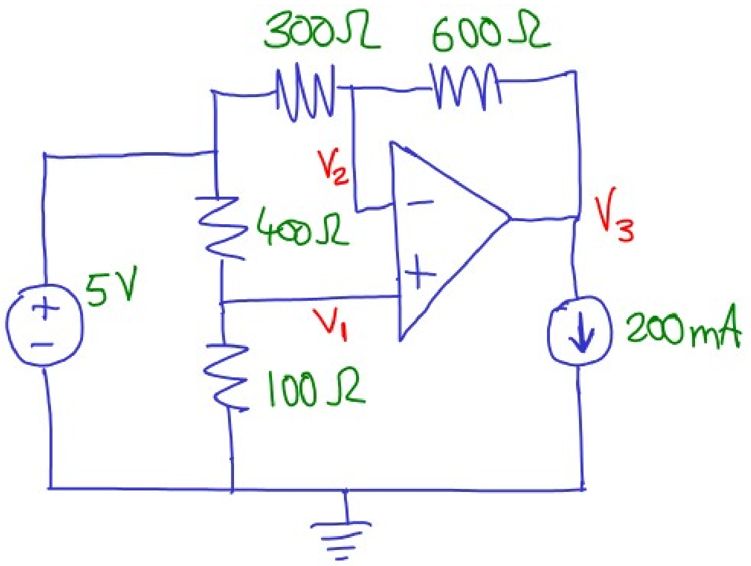
Determine the marked potentials:

a. v_1

b. v_2

c. v_3

d. How much power does the current-source supply to the circuit?



4) [4p]

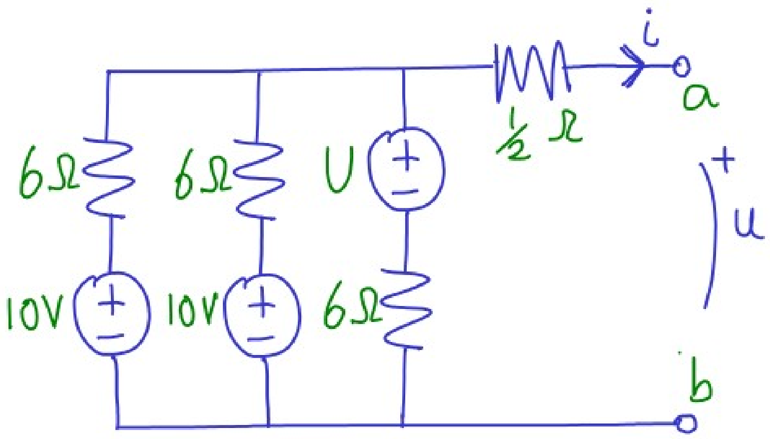
Take $U = 10$ V.

Between the terminals 'a' and 'b':

a. What is the Thevenin resistance of this circuit?

b. How much is the short-circuit current, i_{sc} ?

c. How much is the open-circuit voltage, u_{oc} ?



d. What is the maximum power that can be extracted from the terminals 'a'-'b' ?

e. Now take U = −10 V. What now is the maximum power from the terminals 'a'-'b' ?

End. Please take any remaining time to double-check your solutions!

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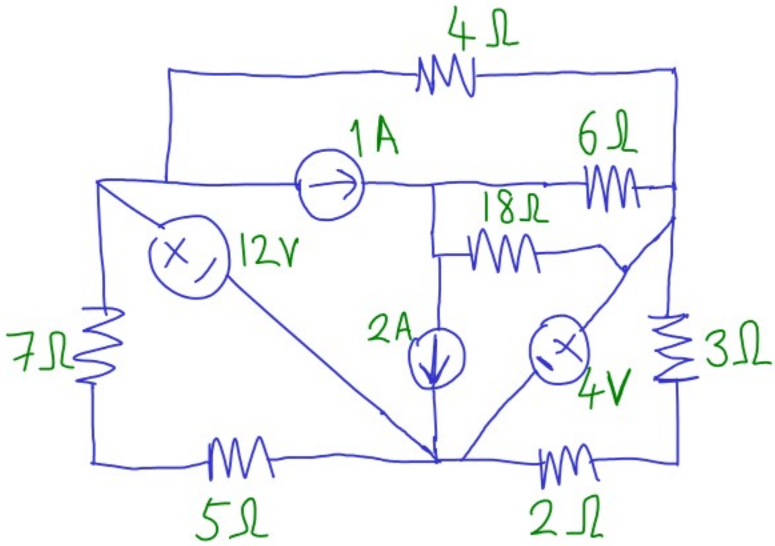
SOLUTIONS.

1) [4p]



a. How much power is supplied to the 5 Ω resistor?

The 7 Ω and 5 Ω resistors are in series, and are connected across the 12 V source. By voltage division this gives 5 V on the 5 Ω resistor. The power is $u^2/R = (5\text{ V})^2 / 5\ \Omega =$ **5 W**.

b. How much power is supplied to the 4 Ω resistor?

We should try to find the current in or the voltage across this resistor. There isn't a clear easy way to find the current. There is however a KVL loop around the two voltage sources and this resistor. Defining the voltage across the resistor as u, KCL gives $12\text{ V} - u - 4\text{ V} = 0$. Thus $u = 8$ V. The direction isn't important as the relation $P = u^2 / R$ for power in a resistor square the voltage and so loses the sign. We get $P = (8\text{ V})^2 / 4\ \Omega =$ **16 W**.

c. How much power is supplied to the 6 Ω resistor?

It might not be entirely obvious, but ... the 6 Ω and 18 Ω resistors are in parallel, and their left end has its current set by the difference between the two current-source currents. KCL at the left of these resistors says that a total of 1 A flows through the pair of resistors from right to left: this is the 2 A source, minus the 1 A source. Current division between the two resistors then puts ¾ of this through the 6 Ω resistor, i.e. 0.75 A.
The power in the resistor is $P = i^2 R = (¾\text{ A})^2 \cdot 6\ \Omega =$ **3.375 W**.
An answer of 3.4 W or even 3.3 W would be acceptable.

d. How much power is supplied from the 4 V source?

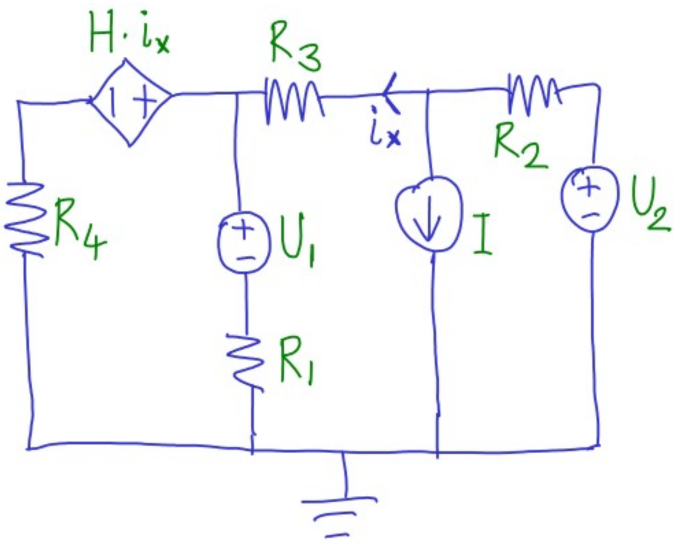
The current *out* from this source's '+' terminal, multiplied by the source's 4 V value, gives the requested output power. By KCL at the '+' terminal, this current is the 1 A flowing in the parallel 18 Ω and 6 Ω resistors (see subquestion 'c'), and the current in the 4 Ω resistor (see subquestion 'b') and the current down the series-connected 3 Ω and 2 Ω resistors.
This is: $i = (2\text{ A} - 1\text{ A}) + (4\text{ V} - 12\text{ V}) / 4\ \Omega + 4\text{ V} / (3\ \Omega + 2\ \Omega) = -0.2\text{ A}$.
The power supplied is $(4\text{ V})(-0.2\text{ A}) =$ **−0.8 W**. So actually 0.8 W goes *in* to the 4 V source.

2) [1p]

a. How many nodes has this circuit, including the reference (earth) node?

6

b. Using 'extended' nodal analysis (wherein no simplifications are made, but a set of equations is written based on a few simple rules), how many equations will be used to represent this circuit?



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5 KCL for all nodes except the reference

3 for the 3 voltage sources, defining relations between node potentials

(and there are 3 unknown currents in these sources, turning up in the KCL equations)

1 for defining the controlling variable of the dependent source, i_x

c. Using the simplifications of 'supernodes', how many KCL equations would be written for this circuit?

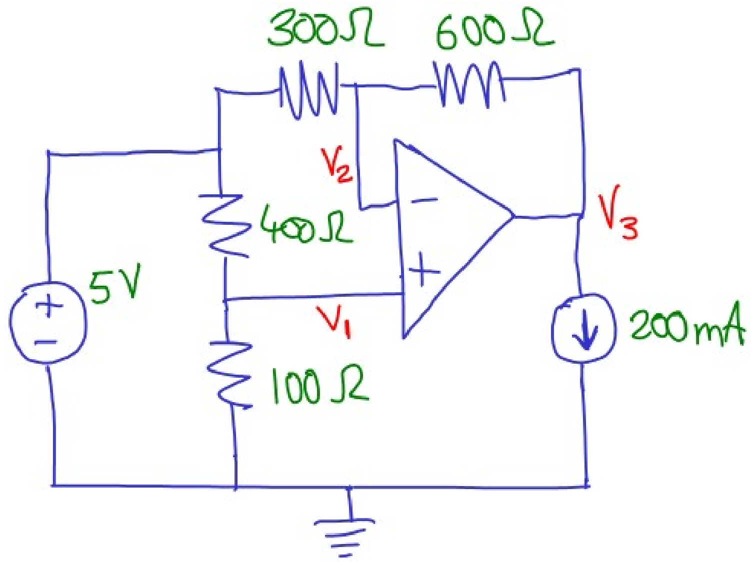
2

the node above U2 becomes part of the 'earth supernode': <u>no KCL here or on the earth node</u>

three nodes (above R4 and R1, and left of R3) become one supernode by the dependent voltage source and the independent voltage source U1 that link these three nodes: this supernode gives one KCL equation

the one remaining node not mentioned above is above the current source: this gives the second KCL equation

3) [3p]



a. v_1

Voltage division of the 5 V source through the 100 Ω and 400 Ω resistors gives **1 V**.

b. v_2

It's an ideal opamp with negative feedback, so $v_2 = v_1 =$ **1 V**.

c. v_3

KCL at v_2: $(v_2 - 5\text{ V}) / 300\ \Omega + 0 + (v_2 - v_3) / 600\ \Omega = 0$.

We already have $v_2 = 1$ V, so this KCL gives a solution of v_3.

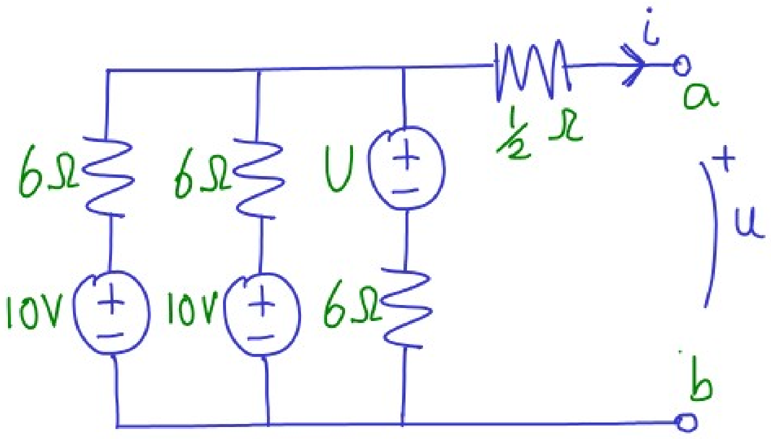
$v_3 = (-4\text{ V} / 300\ \Omega + 1\text{ V} / 600\ \Omega) \cdot 600\ \Omega =$ **−7 V**.

d. How much power does the current-source supply to the circuit?

The current source points from v_3 to 0. The power out from the current source is therefore its current multiplied by $0 - v_3$ (which is $0 - 7\text{ V} = 7\text{ V}$). Therefore: $P = 0.2\text{ A} \cdot 7\text{ V} =$ **1.4 W**.

4) [4p]

$U = 10$ V.
Between the terminals 'a' and 'b':



a. What is the Thevenin resistance of this circuit?

2.5 Ω
Setting all independent sources to zero, we get three 6 Ω resistors in parallel, giving 2 Ω. This equivalent is
then in series with the 0.5 Ω resistor.

b. How much is the short-circuit current, i_{sc} ?

4 A
Several possible approaches (of course!).

By nodal analysis: define the bottom as zero and the top left node as unknown '*v*'; write KCL at '*v*', with 'a'-'b' shorted; solve for *v*, then use Ohm's law to find the current *i*.

Or remove the 0.5 Ω resistor (short it out) and find the Thevenin equivalent of just the three parallel Thevenin-style branches; this is a good deal easier, and then you just need to add the 0.5 Ω to R_T. From the Thevenin equivalent, find the short-circuit current.

Superposition with one source active gives an equivalent 3 Ω for the two other sources' resistors, and this is in parallel with the 0.5 Ω when the terminals are short-circuited. The total current from the active source is then (omitting units for neatness): 10 / (6 + 3·0.5/(3+0.5)) = 1.556 A. A proportion 3/(3+0.5) of this goes through the 0.5 Ω resistor: this is 1.333 A. Becuase all three of the source-branches are the same, there is no need to calculate for the other two sources being active, as these superposition states should also give 1.333 A. So the answer for the short-circuit current from 'a' to 'b' is 3 times 1.333 A = 4 A.

You might even accept the argument that the three left branches are identical in their behaviour between their top and bottom points, and that they together will behave as the same voltage as one such branch (they're in parallel) but 1/3 as much resistance. Then just add the other resistor.

c. How much is the open-circuit voltage, u_{oc} ?

10 V
This can be found by multiplying the two previous answers, or by nodal analysis as described in the solution of part 'b'.

d. What is the maximum power that can be extracted from the terminals 'a'-'b' ?

10 W
The maximum possible power from the terminals of a Thevenin source of U_T and R_T is $P_{max} = U_T^2 /(4\ R_T)$.
$P_{max} = (10\ V)^2 / (4 \cdot 2.5\ \Omega) = 10$ W.

e. Now take U = −10 V. What now is the maximum power from the terminals 'a'-'b' ?

1.11 W
The open-circuit voltage now goes down from 10 V to 3.33 V, but the Thevenin resistance stays the same.
This can be seen by repeating the steps in 'a', 'b' and 'c'. The resulting 1/3 change in U_T makes a 1/9 change in P_{max}.